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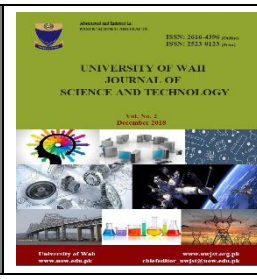

A Study of Different Factors in Single and Multiple Server Queuing Models in Banking System

Sadaf Malik, Naheeda Akhtar, Samra Shoukat and Sajida Javed

Abstract—Queuing theory is discipline of operational research in mathematics. It is mostly dealt with demonstration and analysis of system that deliver service to random demands. In this research work, the queuing numeral, service windows numerals and the maximum service figures are checked out by the help of queuing theory. Queuing models signifies the formation of physical systems. It specifies the server number and their arrangement in order to give services to their clients, and statistical or probabilistic demand nature, by stipulating inconsistency in process of arrival and service. Main concerns of different counters such as "cash deposits counter", "cheque deposits counter" and "cash draw counter" are studied in the field of bank such as arrival and departure of customers and service time provided to them. Distinctive queues and quantities of servers engaged with the procedures are additionally observed by utilizing proper probability distributions. The arrival procedure figured by exponential distributions and administration process is measured by Poisson Distributions. Single server and various server queuing models are utilized in order to dissect lining parameters and execution measures of the framework. WinQsb programming is utilized to perform queuing investigation and queuing simulation in order to process these parameters and execution measures. Moreover, qualities of connection between various execution measures are computed utilizing SPSS (Statistical Package for Social Sciences) 16.

***Index Terms*—Waiting lines, Single Server, Multiple Server, Queuing Model**

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I. Introduction

IN 1930's, Operations Research was used as a subject of science and main subdivision of applied mathematics. It is used to apply adequate analytical methods for assessment creating. Operations Research application can help in making better decisions and solve complicated decision issues. In this research work, an associate optimized model is projected to enhance the bank queuing system supported by queuing theory. This technique will optimize the number of server and improve the service potency that might effectively can cut down service prices and customer's waiting time. One of the extraordinary classes of lining framework that experienced in daily lives, is business benefit framework, in which customers get hold of supplier from business organizations. Many gatherings involve individual-to-individual service at a tough and quick pace, alongside a hair salon, cafeteria, petroleum pump and bank. Numerous business banks have done extraordinary endeavor to blast the bearer proficiency and buyer fulfillment however the most extreme of them are managing a difficult issue of prepared line of clients. In money related organization (i.e. banks), the waiting line of clients appears to be because of low productivity of the lining machine. It emulates the missing of the endeavor reasoning of client driven, and low amenity cost of the device.

Numerous analytical mechanisms used in Islamabad region includes, network analysis, game theory, mathematical logic, queuing theory and simulation etc. But most generally used analytical strategy in numerous field of life is Queuing precept. It is nearly a hundred years old theory. Telephone waiting times for a profitable service in two unlike experiments and consumer thoughts of waiting times considering a psychophysical power function is described in [1]. The negative impact of perceived waiting time on wait evaluations is increased by the pecuniary expenses of waiting. The waiting times are packed in numerous ways like: with music, queuing data, and

knowledge regarding expected waiting time. A supplementary variable technique was introduced by David R. Cox [2]. The basic work of many of the analysis methods of queuing theory is positioned by A.K. Erlang, known as the father of queuing theory for the time period of 1909-1929. He firstly introduced the Poisson process to congestion theory, for the procedure of developing the balance state equations to illustrate the notion of statistical equilibrium mathematically. Allen Flick and Ming Liao obtained the queue time taken by multiple-server queuing system considering arrival and repair rates once these rates are high [3].

Basically there are two types of queuing models; deterministic and probabilistic. In deterministic queuing model, probability distributions is not associated with the arrival of customers and service time, while in a case of probabilistic model, the arrival of events/entities and service time are associated with the probability distribution. For the analysis of non Markovian queues, David G. Kendall introduced the method of Markov chain. Queuing models contradict according to probability distribution of service time, service discipline and consumer arrival rate [4]. Subject to the number of channels, a system of waiting line can be a single server queuing model or multiple servers queuing model. A non-equilibrium queuing system with a finite interval was described by F. Pollaczek [5].

The single server queuing system consists of only one server with multiple or only a single queue. Server provide services to the entities/ customers, which arrive at a system in systematic way in order to be served. M/M/1 queuing model is the basic queuing model where arrival of a customers is according to a Poisson distribution and the service time is exponentially distributed. Arrival rate of a customer is denoted by λ, and a server service rate is denoted by μ, and mean service rate is denoted by 1/μ. FCFS (first come first served) is a basic discipline of serving a single server queuing model. Modeling and analysis of a discrete-time two-phase queuing system is discussed in [6].

Multiple server queuing models consist of two or more than two servers. In a system where servers are identical, if customer arrives and finds that at least one server is free, than a customer can connect to this server to avail service. In a multiple queuing system, single queue and multiple queues are formed. In a case of multiple identical servers and multiples queues jokey behavior exist in a queue, where a customer can be switched to another queue from their queue to get immediate service. The choice of a server does not affect the service, if all servers are identical. FCFS is a basic discipline of a multiple server queuing models. Mansour Tabari et al studied the queuing theory to acknowledge the optimum variety of essential human assets in an instructive establishment distributed in Iran [7]. The queue analysis is performed for various numbers of employee's members.

The main objectives of the current research work are to analyze the waiting lines of banking system by developing the queuing models and measured the performance and utilization factor of the system by analytical methods and queuing simulation technique.

This research paper is organized in six (6) sections. Section II, explains materials and methods. Data collection and analysis for proposed work is done in section III and IV. Section V comprises of interpretation of results. Finally, Section VI concludes the whole research work.

II. Material and Methods

Data for the study is collected for two days per week for a bank located in Islamabad. Three counters in the Bank are utilized for data collection, namely 'Cash Deposit Counter', 'Cash Draw Counter' and Cheque Deposit Counter. The subject is limited to customers and service employees of the bank for the reception counter. Information is gathered by means of 'direct observation', 'questionaries' and 'interviews', while fulfilling the underlined constraints for building a queuing model.

In current research work, two strategies are used. Queuing analysis and Queuing device simulation. Result of Queuing models and Queuing simulation are then compared. Comparison between the only server and multiple server queuing models are additionally created, in step with the steady state probabilities that tends to be estimated through it.

The subsequent assumptions are satisfied by means of records for queuing models.

- Customer's arrival follows a Poisson Probability Distribution.
- Inter-arrival time of consumers are independent and becomes exponentially distributed.
- Service time of the customer is exponentially distributed.
- Purchasers are served by method of any server on first-come first-served basis.
- No customer can depart the queue without obtaining service.
- Queue is incalculable.
- Rate of serving become not obsessed with the queue length. Serving charge remained slight regardless of queue duration.

A. Terminologies and Notations

The following terms and notations are utilized in the model formulation and computation:

P_n = probability of exactly 'n' customers in the system.
N = number of consumers in the system.
L_s = estimated number of customers in the system.
L_q = estimated number of customers in the queue.
W_s = consumer's waiting time in the system.
W_q = consumer's waiting time in the queue.
A_n = average arrival rate of new customers in systems.
μ_n = average service rate for overall systems when n consumers are in systems.

B. Queuing Model for Cash Deposit Counter

The primary queuing model determined during cash deposit counter is multiple server multiple queue model and is measured as M/M/s, wherein M/M represent the Poisson probability distribution of arrival and departure and s (positive integer) denote wide variety of servers. The average time period of a cash deposit counter became approximately 25-30 minutes for every day for a queuing model of cash deposit counter as shown in *Fig. 2*. This model is analyzed by queuing analysis and Queuing simulation.

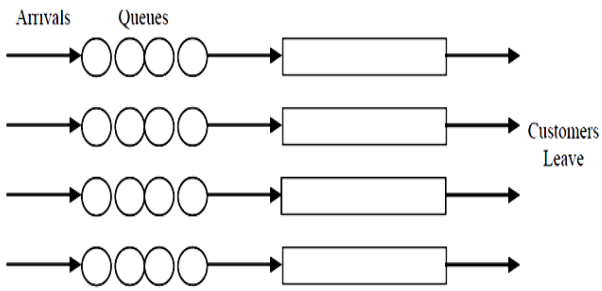

Fig. 1. Queuing Model with multiple queues and multiple parallel servers for cash deposit counter.

C. Queuing Model for Cash Draw Counter

The primary queuing model determined during this counter is multiple server multiple queue model and is measured as M/M/s, wherein M/M represent the Poisson probability distribution of arrival and departure and s (positive integer) denote wide variety of servers. The average time duration of data collection is distinct for every day however it's roughly 20- 25 min for each day for a queuing model for cash draw counter as shown in *Fig. 2*. This model is analyzed by queuing analysis and queuing simulation.

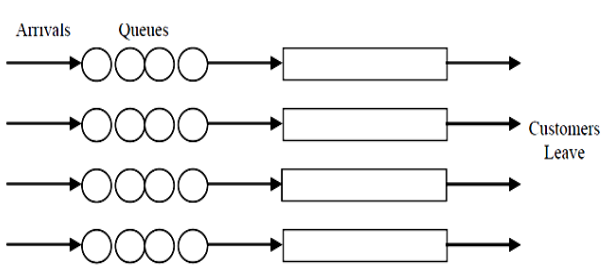

Fig. 2. Queuing model with multiple queues and multiple parallel servers for cash draw counter.

D. Queuing Model for Cheque Deposit Analysis

The primary queuing model with central queuing procedure determined in this counter is single-queue with single-servers, and symbolized as M/M/1, where M/M represents the Poisson chance distribution of arrival and departure and 1 characterize numeral of server that is 1. The time length of cheque deposit counter is 40-45 minutes approximately. The Queuing Model with single queue and single servers is analyzed by queuing analysis as shown in *Fig. 3.*

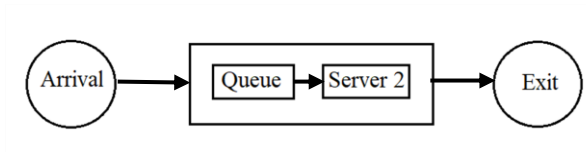

Fig. 3. Queuing Model with single queue and single servers.

E. Parameters of Queuing Models

Parameter of queuing models are,

n = sum of a patrons.

s = figure of parallel channels.

λ = Arrival amount

$$= \frac{1}{average\ no\ of\ customer\ in\ a\ system\ at\ one\ hour}$$

μ = Serving amount

$$= \frac{1}{average\ no\ of\ customer\ served\ per\ hour\ per\ server}$$

s_μ = Rate of serving for s>1 in a system

ρ = Utilization factor

Where,

$$\rho = \frac{\lambda}{s_\mu} \tag{1}$$

ρ in above equation determines service competency being utilized on the common incoming customers.

F. Performance Measures/ Steady State Probabilities

Performance measures are calculated to estimate projected queue time-span of waiting consumers in a queue that is L_q and is calculated by equation (2) given below.

$$L_q = \left[\frac{1}{(s-1)!}\left(\frac{\lambda}{\mu}\right)^s \frac{\mu\lambda}{(\mu s-\lambda)^2}\right] \tag{2}$$

Projected waiting time of the customers in the queue denoted by W_q and can be written as follows.

$$W_q = \frac{L_q}{\lambda} \tag{3}$$

Projected queue time-span of waiting customers in the system is L_s and can be calculated by using following equation.

$$L_s = L_q + \left(\frac{\lambda}{\mu}\right) \tag{4}$$

Projected waiting time of the consumers in the system which is W_s and can be measured as in equation (5) below.

$$W_s = \frac{L_s}{\lambda} \quad (5)$$

III. Data Collection

In this research work, occurrence of events such as, customer's arrival, a beginning of a service in a counter and end of service in counters. Two days data is collected from one of the banks of Islamabad. The methods engaged during data collection are "direct observation", "personal interrogated" and feedback form "administering" by the investigator. Service time is recorded through stop watch. Number of consumers are different for 3 counters in both days. After collection of data, it is tabulated in a spread sheets and then analysis is carried out.

Two techniques Queuing analysis and Queuing system simulation are used in the current research work. Result of Queuing models and Queuing simulation are compared. Comparison between the single server and multiple server queuing models is also made, according to the steady state probabilities that are estimated through it. Simulation is used to create a model. Arrival rate and service time are random number generated for each entity to fit distribution, which is determined by study time of original system. Simulation and queuing model both have their own advantages. Queuing models are simpler and easy to apply on data and provide more standard results. Once the user create a model and then validates and verifies it, then one can easily analyze the response and adjust the system.

When the model M/M/C is with multiple queues, it means multiple servers with multiple queues and the number of customers are infinite, so the solution is difficult to obtain. So when μ is difficult to obtain through analytical method a Monte Carlo simulation is used. It is a discrete event simulation and provides a very reliable answer as in this case when simulation is run for more than thousand times. In banks "cash deposit" and "cash draw" counters composed multiple server with multiple queues. Queuing simulation is used in such situation to measure the performance of an individual as well as overall system. Simulation is performed using computer based software WinQsb.

IV. Analysis of Data

In this research work, WinQsb software is used to predict the evaluation of queuing analysis and queuing simulation of single server and multiple servers queuing model at bank using arrival rate (λ), service rate (μ), and number of server. Queuing simulation is performed for cash deposit and cash draw counter.

V. Interpretation

Tables I and II shows the results of queuing analysis and queuing simulation. The result of queuing analysis shows that for m/m/2 model, the waiting time of customer in cash deposit is 4.5 minutes, for cash draw the waiting time is 5 minute and waiting time of cheque deposit counter is 2.5 minutes. As in cash draw and cash deposit counter jokey behavior exists so queuing simulation is also performed for these counter because queuing simulation clearly evaluate the performance of multiple servers multiple queues. In cash deposit counter, in queue 2 each customer has to wait for 27.9 minute and in queue 1 each customer has to wait for 20.8 minute. In cash draw counter, in queue 2 each customer has to wait for 32 minute and in queue 1 each customer has to wait for 28 minute.

TABLE I
PARAMETERS AND OVERALL PERFORMANCE MEASURES OF QUEUING MODEL USING QUEUING ANALYSIS FOR 1ST DAY

Performance Measures	Cash Deposit Counter		Cash Draw Counter		Cheque Deposit Counter
	m/m/2	m/m/1	m/m/2	m/m/1	m/m/1
Arrival rate(λ) per hour	96.91	96.91	72	72	49.7
Service rate (μ) per hour	54.5	54.5	41.09	41.09	67.9
Utilization factor (ρ)	88%	88%	87%	87%	73%
Average number of customer in the system(L_s)	8.48	7	7.66	7.2	2.7
Average number of customer in the queue(L_q)	6.7	6.12	5.9	6.3	1.9
Average time customers spends in the system (W_s)	0.08	0.1653	0.1065	0.2	0.0549
Average time customers spends in the queue (W_q)	0.0692	0.1492	0.0821	0.1756	0.040

Tables III and IV shows the results that for m/m/2 model, the waiting time of customer in cash deposit is 2.12 minute, for cash draw the waiting time is 9.09 minute and waiting time of cheque deposit counter is 2.25 minute. Queuing simulation clearly evaluate the performance of cash draw and cash deposit counter as these counter have multiple servers multiple queues behavior. In cash deposit counter, in queue 2 each customer has to wait for 27.67 minute and in queue 1 each customer has to wait for 20 minute. In cash draw counter, in queue 2 each customer has to wait for 30 minute and in queue 1 each customer has to wait for 23.3 minute.

A. Correlation between L_q, W_q and ρ

Results of queuing analysis show that the value of utilization factor varies as the waiting time of customers and average number of customers increases or decreases. To analyze this behavior, a spear's man correlation coefficient by using SPSS is used.

Results of Table V clearly shows that positive correlation exist between factors of performance measure. It is observed that there exists a strong correlation between ρ and L_q as compared to the correlation which occurs between ρ and W_q. It is proved that when queue length increase ρ also increase rapidly, by increasing the number of servers, ρ inevitably reduces.

TABLE II
PARAMETERS AND OVERALL PERFORMANCE MEASURES OF QUEUING MODEL
THE USAGE OF QUEUING SIMULATION FOR 1st DAY

Performance Measures	Cash Deposit Counter			Cash Draw Counter		
	*S 1	*S 2	S 1 + S 2	S 1	S 2	S 1 + S 2
Mean inter-arrival time (min)	0.66	0.66	0.66	0.76	0.76	0.76
Mean Serving time (min)	1.1	1.1	1.1	1.46	1.46	1.46
Server utilization	99%	99%	99%	99.28%	99.28%	99.28%
Average number of customers in the queue (L_q)	32.7	44.1	76.96	33.6	45.6	79
Average time customers spends in the queue (W_q)	20.8	27.9	24.4	24.8	32	28

*S 1 = Server 1, *S 2 = Server 2

VI. CONCLUSION

The queuing distinctiveness at the bank of Islamabad is analyzed using queuing evaluation and queuing simulation in three different counters. Result of evaluation show that waiting time, average queue length of customers as well as utilization of servers could be reduced when the multiple servers queuing model is used as compared to the single server queuing model. Strength of correlation between three different performance measures are calculated and it is observed that the strong correlation exist between ρ and L_q. It is therefore needed to use optimal number of servers to reduce the queue length. This research work will help banks to increase their quality of services (QOS), by forestalling, in a case when there are more customers in a system. Hence, presented work will not be only helpful for under study departments but also for all those fields of life having waiting line phenomena such as bank, post offices, railway ticketing, airlines, sales checkout etc.

TABLE III
PARAMETERS AND OVERALL PERFORMANCE MEASURES OF QUEUING MODEL
THE USAGE OF QUEUING ANALYSIS FOR 2nd DAY.

Performance Measures	Cash Deposit Counter		Cash Draw Counter		Cheque Deposit Counter
	m/m/2	m/m/1	m/m/2	m/m/1	m/m/1
Arrival rate(λ) per hour	79.41	79.41	75.8	75.8	50
Service rate (μ) per hour	49.58	49.58	41.3	41.3	69.2
Utilization factor (ρ)	79%	79%	91%	91%	72%
Average number of customer in the system (L_s)	4.39	3.98	11.6	11.13	2.6
Average number of customer in the queue (L_q)	2.79	3.18	9.79	10.22	1.8
Average time customers spends in the system (W_s)	0.0556	0.1007	0.1543	0.2941	0.0521
Average time customers spends in the queue (W_q)	0.0354	0.0422	0.1516	0.2699	0.0376

This research work will also help to analyze an existing system, and for the improvement of the next system because now banks can calculate the number of waiting customers in a queue, and the number of going away customers in each day. In each day by computing the no of outgoing and coming customers, now banks can set a goal, that how many servers will be required in a main branch or a small branch of a bank to serve customers. Presented research work can also be utilized for the new installations of ATM machines.

TABLE IV
PARAMETERS AND OVERALL PERFORMANCE MEASURES OF QUEUING MODEL
THE USAGE OF QUEUING SIMULATION FOR 2nd DAY

Performance measures	Cash deposit counter			Cash draw counter		
	*S 1	*S 2	S 1 + S 2	S 1	S 2	S 1 + S 2
Mean inter-arrival time (min)	0.76	0.76	0.76	0.8	0.8	0.8
Mean Serving time (min)	1.28	1.28	1.28	1.4	1.45	1.45
Server utilization %	99.22	99.22	99.22	98.6	98.6	98.6
Average number of customers in the queue (Lq)	27.7	39.3	67.1	29	41.8	71.8
Average time customers spends in the queue (Wq)	20	27.67	23.8	23	30.8	27.01

*S 1 = Server 1, *S 2 = Server 2

TABLE V
STRENGTH OF ASSOCIATION BETWEEN ρ, L_q AND W_q

Type of Correlation	Utilization Factor (ρ)	No. of customers waiting in a queue (L_q)	Customer waiting time in a queue (W_q)
Pearson Correlation	1	.926	.899
Sig. (2-tailed)	0.000	.000	.000
Pearson Correlation	.926	1	.982
Sig. (2-tailed)	.000	.000	.000
Pearson Correlation	.899	.982	1
Sig. (2-tailed)	.000	.000	.000

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