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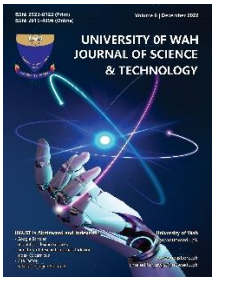

Catalogue of Efficient Repeated Measurements Designs for $p_3 = 5, 6, 7$

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Abstract
Minimal strongly balanced repeated measurements designs are economical to (i) balance the residual effects, and (ii) estimate the treatment effects and residual effects independently. The situations where these minimal designs cannot be constructed, minimal nearly strongly balanced repeated measurements designs are used. In this article, some efficient minimal circular nearly strongly balanced repeated measurements designs are constructed in periods of three different sizes with the smallest of sizes 5, 6, and 7.

Keywords
Carry over effects; Residual effects; Strongly balanced RMDs; Weakly balanced RMDs; Minimal designs.

1. Introduction

With the use of repeated measurements designs (RMDs) residual effects may arise. Effect which a treatment has during its period of application (treatment effect) may persist into the preceding period. Such effect is called residual effect or carry over effect. Minimal balanced or strongly balanced RMDs are useful to balance these effects economically. RMD will be minimal balanced for residual effects if each treatment is immediately preceded once by each other treatment (excluding itself). RMD will be minimal strongly balanced (SBRMD) if each treatment is immediately preceded once by each other treatment (including itself). Using method of cyclic shifts, these designs can be constructed only through Rule I in circular formation. Rule II provides minimal circular nearly strongly balanced RMDs (MCNSBRMDs) for most of the remaining cases. RMDs

in which each treatment appears with all other treatments (including itself) exactly once as preceding treatments except v-1 which does not appear with itself are called MCNSBRMDs.

[1,2] first initiated RMDs for $v = p$. [3] introduced the idea of a CBRMD. [4] constructed balanced and strongly balanced minimal RMDs with unequal period sizes. Using method of cyclic shifts, [5] constructed (i) efficient RMDs with equal and unequal period sizes, (ii) SBRMDs for periods of two different sizes for $5 \leq v \leq 10$, $3 \leq p_1 \leq 6$ and $3 \leq p_2 \leq 10$. [6] constructed universally optimal weakly balanced RMDs for $p = v$. [7] and [8] developed some generators to obtain circular balanced and weakly balanced RMDs respectively in periods of equal sizes. [9] developed some infinite series to obtain minimal circular strongly partially BRMDs in periods of equal sizes. [10] developed generators to obtain MCSBRMDs for p_1 even, $3 \leq p_2 \leq 10$ and $2 \leq p_2 \leq 9$. [11] developed some generators for general construction of MCSBRMDs in periods of unequal sizes. Recently, [12] constructed MCNSBRMDs in periods of three different sizes with smallest of size two. Here, MCNSBRMDs are constructed for $v \leq 100$, $7 \leq p_1$ (odd) ≤ 9, $6 \leq p_2 \leq 8$, $5 \leq p_2 \leq 7$.

Efficiency for residual effects and of separability are discussed in Section 2. In Section 3, MCNSBRMDs are constructed along with their efficiencies in periods of three different sizes when the $p_3 = 5$, 6, and 7.

2. Efficiency for Residual Effects and of Separability

In this Section, procedures are explained to find the efficiency for residual effects and efficiency of separability.

2.1 Efficiency for residual effects (*Er*)

[13,14] expressed that the canonical efficiency factors are the harmonic mean of non-zero Eigen values of their respective C^* (information matrix). Design possessing high value of E_r will be efficient to estimate the residual effects.

2.2. Efficiency of separability (*Es*)

Following formula is for *Es* of MCNSBRMDs derived from the relation given by [15].

$$Es = \left[1 - \left\{ \frac{(l_1 + 4l_2)v - (l_1 + 2l_2)^2}{(v-1)(l_1 + 2l_2)^2} \right\}^{\frac{1}{2}} \right] \times 100\%,$$

where l_1 and l_2 are the number of treatments preceded once, and two respectively by other treatment. *Es* of proposed designs is at least 97%, therefore, these designs are highly efficient according to this criterion.

3. Methodology

In this Section, using method of cyclic shifts (Rule II) introduced by [16], MCNSBRMDs are constructed using following sets in periods of three different sizes for $v = ri+s+2$, here $p_1 = r$ (odd), $p_2 = s$, and $p_3 = u$. In these designs only one pair (v-1, v-1) do not appear while all others appear once. $S_z = \left[q_{z1}, q_{z2}, ..., q_{z(r-1)} \right]; \qquad z = 1, 2, ..., i.$

$$S_{z+1} = \left[q_{(z+1)1}, q_{(z+1)2}, ..., q_{(z+1)(s-1)} \right]$$

$$S_{z+2} = \left[q_{(z+2)1}, q_{(z+2)2}, ..., q_{(z+2)(u-2)} \right] t,$$

where

- Each elements of sets lies between 0 and v-2.
- S^* contains each of 0 to v-2 exactly once.
- S^* contains (i) each value of all sets, (ii) complement of the sum of values in each set of S_z and S_{z+1}, where complement of 'a' is v-1-a.

Example 3.1: S_1 = [1,2,3,15,5,6], S_2 = [8,9,10,11,12], S_3 = [4,7,13]t provide following MCNSBRMD for $v = 17$ in $p_1 = 7$, $p_2 = 6$ and $p_3 = 5$.

Periods	**Subjects**															
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	0
3	3	4	5	6	7	8	9	10	11	12	13	14	15	0	1	2
4	6	7	8	9	10	11	12	13	14	15	0	1	2	3	4	5
5	5	6	7	8	9	10	11	12	13	14	15	0	1	2	3	4
6	10	11	12	13	14	15	0	1	2	3	4	5	6	7	8	9
7	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Periods	**Subjects**															
	17	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6	7
3	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	0
4	11	12	13	14	15	0	1	2	3	4	5	6	7	8	9	10
5	6	7	8	9	10	11	12	13	14	15	0	1	2	3	4	5
6	2	3	4	5	6	7	8	9	10	11	12	13	14	15	0	1
Periods	**Subjects**															
	33	**34**	**35**	**36**	**37**	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

2	4	5	6	7	8	9	10	11	12	13	14	15	0	1	2	3
3	11	12	13	14	15	0	1	2	3	4	5	6	7	8	9	10
4	8	9	10	11	12	13	14	15	0	1	2	3	4	5	6	7
5	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16

Catalogues of these designs are presented for $v \leq 100$, $7 \leq p_1 \leq 9$, $6 \leq p_2 \leq 8$ in Appendix as Table 1, Table 2, and Table 3 for $p_3 = 5$, 6 and 7 respectively.

Apppendix

Table 1: MCNSBRMDs for $v \leq 100$, $7 \leq p_1$ (odd) ≤ 9, $6 \leq p_2 \leq 8$ and $p_3 = 5$, where $p_1 > p_2$.

v	p_1	p_2	p_3	**Sets of Shifts**	***Es***	***Er***
17	7	6	5	[1,2,3,15,5,6]+[8,9,10,11,12]+[13,7,4]t	0.99	0.93
31	7	6	5	[10,2,3,4,5,6]+[8,9,17,11,12,13]+[14,15,16,1,18,19]+ [22,23,24,25,27]+[26,28,21]t	0.99	0.89
45	7	6	5	[1,25,3,4,5,6]+[26,9,10,11,12,13]+[15,16,30,18,19,20]+ [22,23,24,2,8,32]+[29,17,36,43,33,34]+[31,37,38,39,40]+ [41,42,27]t	0.99	0.88
59	7	6	5	[26,2,3,4,5,6]+[0,8,9,10,11,13]+[23,15,17,18,25,43]+ [29,16,24,19,38,27]+[22,37,31,55,14,34]+[36,30,1,20,47,44]+ [28,41,45,46,40,48]+[50,51,52,53,35]+[32,56,57]t	0.99	0.87
73	7	6	5	[52,2,3,4,5,6]+[8,9,10,11,12,15]+[40,16,17,18,19,20]+ [23,70,24,25,26,27]+[13,42,31,32,33,37]+[36,34,30,47,29,44]+ [43,41,45,46,50,1]+[39,51,48,53,54,58]+[49,63,69,60,64,71]+ [22,65,61,67,35]+[59,66,55]t	0.99	0.87
19	9	6	5	[1,3,2,4,5,6,8,7]+[11,10,12,13,17]+[15,16,14]t	0.99	0.94
37	9	6	5	[1,2,3,4,5,6,35,16]+[11,10,12,13,14,15,17,7]+ [19,20,21,22,23,24,8,25]+[28,29,33,31,32]+[30,34,26]t	0.99	0.91
55	9	6	5	[51,1,4,53,5,6,52,44]+[10,11,12,13,20,15,16,2]+ [19,14,22,21,23,24,25,50]+[28,29,30,31,32,33,34,26]+ [37,38,39,40,41,43,42,8]+[46,47,48,49,35]+[17,7,3]t	0.99	0.90
73	9	6	5	[1,38,13,3,5,6,69,9]+[10,11,12,4,14,24,26,18]+ [29,19,21,22,23,15,2,68]+[27,28,20,58,31,32,33,43]+ [37,7,47,71,41,42,34,45]+[46,49,48,39,50,51,52,53]+ [55,57,56,40,59,60,61,62]+[63,64,65,66,67]+[8,70,30]t	0.99	0.90
91	9	6	5	[55,2,3,4,5,6,7,8]+[10,11,12,13,14,15,16,17]+ [19,21,20,22,23,24,25,28]+[29,26,39,73,32,63,34,37]+ [35,38,49,30,41,42,43,46]+[44,47,48,40,59,60,54,53]+ [1,56,57,58,50,51,61,64]+[62,66,65,67,68,69,70,74]+ [82,71,75,76,77,81,79,80]+[31,83,87,85,86]+[84,18,33]t	0.99	0.90
29	9	7	5	[9,2,3,16,5,6,7,8]+[18,12,11,13,14,24,15,4]+ [27,20,21,22,23,17]+[25,26,19]t	0.99	0.92
47	9	7	5	[1,2,3,4,5,6,17,8]+[41,11,12,13,14,15,16,7]+ [19,20,21,22,23,10,25,26]+[28,29,30,31,32,33,34,35]+ [37,38,39,40,44,42]+[43,27,45]t	0.99	0.91
65	9	7	5	[29,2,3,4,5,6,7,8]+[10,11,18,13,14,15,16,22]+ [19,20,21,17,23,25,24,31]+[28,1,40,26,32,33,34,35]+ [63,38,39,30,41,42,43,52]+[46,47,48,49,50,51,44,53]+ [55,56,57,58,59,45]+[61,62,37]t	0.99	0.90

83	9	7	5	[1,2,3,4,5,6,7,54]+[10,11,47,13,61,62,16,17]+ [19,20,21,22,23,24,25,58]+[28,29,30,31,15,33,18,35]+ [37,38,39,8,41,42,43,44]+[81,12,48,49,50,51,52,53]+ [55,56,57,26,59,60,78,32]+[64,65,66,67,68,40,70,71]+ [73,74,75,76,77,45]+[79,80,46]t	0.99	0.87
21	9	8	5	[1,2,3,4,5,10,7,8]+[6,12,11,13,15,18,16]+[17,14,19]t	0.99	0.94
39	9	8	5	[26,1,3,4,5,9,8,20]+[10,11,12,13,14,15,16,17]+ [19,7,21,22,23,37,25,2]+[28,29,31,30,32,33,18]+[35,36,24]t	0.99	0.92
57	9	8	5	[1,2,3,4,25,6,7,8]+[5,11,12,13,14,15,16,17]+ [19,44,21,22,23,10,39,28]+[26,29,31,30,32,33,34,38]+ [37,35,24,55,41,42,20,46]+[43,54,48,49,50,51,52]+[53,47,40]t	0.99	0.91
75	9	8	5	[39,2,3,4,5,6,7,8]+[10,11,12,44,14,15,16,17]+ [19,20,21,22,23,24,25,26]+[65,37,30,31,32,33,34,35]+ [71,38,70,18,40,41,43,13]+[46,47,48,49,50,51,55,53]+ [52,1,64,58,59,60,61,62]+[57,72,66,67,68,69,56]+[29,54,28]t	0.99	0.90
93	9	8	5	[1,2,3,4,61,6,7,8]+[10,11,12,9,14,15,16,17]+ [19,24,21,22,23,33,28,26]+[25,29,30,31,32,13,18,27]+ [37,38,39,48,41,42,43,44]+[46,47,35,49,82,51,52,53]+ [55,56,57,58,59,60,91,62]+[64,65,66,67,68,69,70,20]+ [73,74,75,76,77,78,79,40]+[50,83,84,85,86,87,88]+[89,90,5]t	0.98	0.88

Table 2: MCNSBRMDs for $v \leq 100$, $p_1 = 9$, $7 \leq p_2 \leq 8$ and $p_3 = 6$.

v	***p*₁**	***p*₂**	***p*₃**	**Sets of Shifts**	***Es***	***Er***
21	9	7	6	[2,1,3,4,5,6,7,12]+[10,11,8,13,14,15]+[16,17,18,19]t	0.99	0.85
39	9	7	6	[1,2,3,4,5,6,7,10]+[8,11,20,33,14,15,16,26]+ [19,12,21,22,23,24,34,17]+[28,29,31,30,32,13]+ [25,35,36,37]t	0.99	0.92
57	9	7	6	[1,2,3,4,25,6,7,8]+[51,11,12,13,14,15,26,17]+ [19,20,21,22,23,24,5,16]+[28,29,30,31,32,33,34,36]+ [55,38,39,40,41,42,43,44]+[46,47,48,49,35,10]+ [52,53,54,37]t	0.99	0.91
75	9	7	6	[1,2,3,42,5,6,7,8]+[10,11,12,13,14,15,16,17]+ [19,44,21,22,23,24,25,26]+[28,29,47,31,32,33,34,35]+ [37,38,39,9,41,4,43,49]+[46,66,20,30,54,51,52,53]+ [55,56,57,58,59,60,61,62]+[64,65,48,67,68,69]+ [70,71,45,73]t	0.89	0.97
93	9	7	6	[1,2,3,60,5,6,7,8]+[77,11,12,13,14,15,16,17]+ [71,19,21,22,23,27,49,26]+[28,29,30,31,32,33,34,35]+ [37,38,39,40,41,42,43,52]+[10,47,48,25,86,66,44,89]+ [55,56,51,58,59,4,61,62]+[64,65,57,67,68,69,79,20]+ [73,74,75,76,46,78,70,80]+[82,83,84,85,50,87]+[88,53,90,91]t	0.99	0.90
31	9	8	6	[25,2,4,3,5,6,7,8]+[10,12,11,13,15,14,20,16]+ [17,19,21,22,24,28,1]+[26,27,23,29]t	0.99	0.93
49	9	8	6	[1,2,3,4,5,6,7,20]+[10,11,12,13,15,14,16,17]+ [19,8,35,22,9,24,25,32]+[28,29,44,31,26,33,34,21]+ [37,38,39,40,41,27,43]+[30,45,46,47]t	0.99	0.91
67	9	8	6	[1,2,3,4,35,6,7,8]+[10,11,12,13,14,30,16,17]+ [19,20,21,22,24,23,25,26]+[28,29,45,32,31,34,33,5]+ [53,37,39,40,41,42,43,65]+[46,48,47,49,50,51,52,38]+ [55,56,57,58,59,60,63]+[62,61,64,44]t	0.99	0.91
85	9	8	6	[1,50,3,4,5,6,7,8]+[10,11,12,13,14,15,16,17]+ [19,74,21,22,23,24,83,52]+[28,2,30,31,32,33,34,35]+ [37,38,39,40,41,42,43,20]+[46,47,48,49,29,51,26,79]+ [55,56,58,57,59,9,61,80]+[64,65,66,67,68,54,70,71]+ [73,77,75,76,44,25,72]+[53,81,82,78]t	0.99	0.90

Table 3: MCNSBRMDs for $v \leq 100$, $p_1 = 9$, $p_2 = 8$ and $p_3 = 7$.

v	p_1	p_2	p_3	**Sets of Shifts**	***Es***	***Er***
32	9	8	7	[1,2,3,4,6,5,25,16]+[10,11,12,13,30,15,7,17]+ [19,20,21,22,23,24,8]+[26,27,28,29,14]t	0.99	0.87
50	9	8	7	[35,26,3,4,5,10,7,8]+[6,11,12,13,48,15,16,17]+ [19,20,43,23,22,24,2,25]+[28,29,30,31,32,33,34,1]+ [37,38,39,40,41,42,21]+[44,45,46,47,14]t	0.99	0.87
68	9	8	7	[1,3,40,64,5,6,7,8]+[50,10,4,13,14,60,24,17]+ [30,20,21,22,16,23,25,26]+[28,29,19,32,31,33,35,34]+ [37,38,39,15,41,42,43,44]+[46,47,48,49,51,11,52,53]+ [55,56,57,58,59,2,61]+[62,63,12,65,66]t	0.99	0.88
86	9	8	7	[26,2,3,4,5,30,7,8]+[80,40,12,13,14,15,16,19]+ [17,11,21,22,23,24,25,9]+[28,29,6,31,32,33,34,35]+ [37,38,39,41,20,42,44,43]+[1,47,60,49,67,51,53,52]+ [55,56,57,58,59,48,61,62]+[64,65,66,50,68,69,79,71]+ [73,74,75,76,77,78,70]+[10,81,82,83,84]t	0.99	0.88

4. Results and Discussion

Because their Es is at least 97 percent, the designs we've proposed are extremely effective for controlling the residual effect and estimating the direct effect independently. As a result, these designs are the most suitable choices for situations in which MCSBRMDs cannot be developed. Created catalogues are valuable for the experimenters to apply their preferred designs.

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