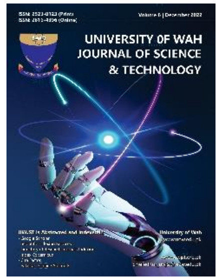
UW Journal of Science and Technology Vol. 7, Issue, 1 (2023) 51-75
ISSN: 2523-0123 (Print) 2616-4396 (Online)

University of Wah
Journal of Science and Technology

www.uow.edu.pk



New analytical wave structures for some nonlinear dynamical models via mathematical technique

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Abstract
In this paper, we used Yu-Toda-Sasa-Fukuyama (YTSF) model, the (2 + 1)-dimensional nonlinear evolution equation, and water wave propagation with surface tension to find new traveling wave solutions by applying $\exp(-\phi(\eta))$expansion method (EEM). The proposed nonlinear wave models are essential in coastal and offshore studies to understand wave propagation, wave transformation, and coastal processes. They also play a crucial role in cryptography, the regulation of heartbeats, and mathematical physics. We generate 3D, 2D, and contour plots of discovered solutions by selecting suitable values for arbitrary parameters within the accurate range space. Hyperbolic, trigonometric, and exponential functions are used to express the resulting traveling wave solutions. The received solutions included dark, bright, periodic, kink, singular, bell-type, hyperbolic solitary wave solutions, and many more. By changing model parameters, it is possible to change the dynamics of the solutions that the model generates. These results highlight the complexity and nonlinear behavior of the system, indicating the need for further analysis and providing valuable insights for understanding and modeling similar physical systems. This work breaks new ground by utilizing the EEM to uncover solitonic solutions for an unsolved model, pushing the boundaries of the existing literature by introducing a new mathematical technique for addressing fractional nonlinear physical models. The proposed method is brief, clear, and trustworthy, resulting in fewer calculations and broad application.

Keywords
Solitons; $\exp(-\phi(\eta))$-expansion method; Yu-Toda-Sasa-Fukuyama model; (2 + 1)-dimensional nonlinear evolution equation; Water wave propagation with surface tension; hyperbolic solitary wave solution

1. Introduction

Nonlinear system theory has applications in many fields, including population growth [1], heartbeat regulation [2], cyber security [3], image encryption [4], and many more [5–9]. Due to the multiple applications of nonlinear evolution equations (NLEEs), many researchers have developed an interest in solving these types of equations. The study of nonlinear partial differential equations (NLPDEs) is a very competitive and active subject of research in the areas of theoretical physics, applied mathematics, and numerous engineering applications [10–12]. To understand physical phenomena, it is necessary to obtain the outcomes of the governing NLPDEs. In the presence of noise or random events, PDEs are appropriate mathematical models for modeling complex systems. The research provides a significant contribution by looking at a wide variety of soliton solutions that cover different wave forms. Solitons are non-dispersive, self-reinforcing wave packets that keep their form and speed while they move across a medium. Solitons are significant because they are present in almost every scientific field. Solitons are fundamental components of several physics disciplines, including nonlinear optics, condensed matter physics, and plasma physics. Additionally, they have uses in biology, engineering, and telecommunications [13–16]. Understanding and characterizing various solitons, such as dark solitons, bright solitons, kink solitons, and rogue waves, among others, offers prospects for technological improvement as well as useful insights into the behavior of complex systems [17]. This work advances knowledge of soliton dynamics and its applications in various fields by examining a broad spectrum of soliton solutions. Several unique methods have been devised to secure their exact and approximate solutions, allowing us to perform qualitative and quantitative analyses of these NLPDEs. These methods have been introduced during the past few decades, including the power series expansion technique [18], the exp-function methods [19–21], transformed rational function method [22], homotopy perturbation method [23], variational method [24], the improved F-expansion method [25, 26], direct algebraic method [27], the novel Kudryashov method [28–30], the modified Khater method [31], the extended Fan-expansion method [32], he’s variational iteration methods [33] and many others [34–36].

The YTSFE is a modification of the Bogoyavlenskii-Schif equation. In 2011, Darvishi [37] found a number of closed-form solutions to the (3+1)-dimensional potential YTSFE by employing the modified extended homo-clinic test technique. In 2014, researchers Hu et al. [38] employed the three-wave method to discover fresh kink multi soliton solutions pertaining to the potential YTSF equation. Wei Tan employed the extended homo-clinic test technique [39] in 2016 to get precise kinky breather-wave solutions. In 2017, Roshid [40] utilized the lump solution method to develop an exact solution for the YTSF equation, aiming to explore its potential for further study. In 2019, Zhao and He [41] utilized the bilinear method as a means to investigate the potential-YTSF equation in fluid dynamics within a dimensional context. In 2022, Abdel-Gawad [42] used the power series expansion method and the Lie symmetry analysis methodology to provide accurate analytical solutions to the YTSFE. Additionally, Q. H. Zhu and J. M. Qi employed this equation in 2022 to obtain elliptic solutions and hyperbolic function solutions [43]. Numerous research teams have examined the solution of (2 + 1)-dimensional NLEEs through the application of analytical and numerical techniques. A. Bekir employed the tanh-coth method [44] as an analytical approach, while the sine-cosine method [45] was also utilized to solve NLEEs. Two variants of the (2 + 1)-dimensional Boussinesq-type equations with positive and negative exponents were explored by Feng et al. [46] for their bifurcations and overall dynamic behavior. The generalized auxiliary equation method has been successfully applied to the (2+1)-dimensional soliton equations by Zhang [47]. Using the Hirota method, Zhang et al. [48] investigated the double-lump solution for two-mode optical fiber. Chen and Liu created a unified Kadomtsev-Petviashvili equation for surface and interfacial waves moving in a rotating channel. Das et al. [49] recently conducted a study focusing on the investigation of oblique scattering of surface waves by a thick partially immersed rectangular barrier. Recently, long-time survival results [50, 51] for gravity water waves of infinite depth have just been published. The proposed models give trustworthy and clear solutions by using the EEM.

In this article, we present an EEM for directly discovering traveling wave solutions to nonlinear PDEs. The robustness and effectiveness of the EEM can vary depending on factors such as the complexity of the equations, the specific problem domain, and the underlying assumptions of the method. This mathematical technique has found applications in diverse scientific disciplines for obtaining solutions to both NLEEs and NLPDEs. J. Ahmad et al. used the EEM to study the soliton solutions of the Caudrey-Dodd-Gibbon equation [52]. In 2018, Jalil Manafian employed nonlinear Boussinesq equations to generate some new traveling and periodic solitary waves [53]. In 2021, Haci Mehmet Baskonus et al. applied the EEM to develop dark and singular soliton solutions for the Chen-Lee-Liu equation [54]. Pankaj et al. [55] applied the EEM to soliton solutions of the nonlinear Schrodinger system. We apply the proposed method to certain NLPDEs, such as the YTSF model [56, 57], the (2+1)-dimensional NLEEs [58, 59], and the water wave propagation with surface tension [60, 61]. Wave transformation can be used to turn the NLPDEs into nonlinear ordinary differential equations (NLODEs) and then obtain closed-form solutions to these equations. Further analysis and comparison of the different methods, taking into account factors like accuracy, convergence properties, computational efficiency, and applicability to various problem domains, would be needed to determine the effectiveness of the EEM relative to other existing approaches. Real-world applications such as bacterial growth and compound interest are frequently represented using the EEM. The EEM is commonly used in the biological sciences to estimate the amount of a certain quantity over time, such as population size.

Based on the existing literature, research gaps in the study of the proposed models may be found. While various methods were used to find exact solutions for the presented models, there are still areas that need to be investigated further. Exploring the use of different solution methods, such as the inverse scattering transform [62], Darboux transformation [63], or Backlund transformation [64], to produce novel exact solutions for the proposed models is one potential direction of investigation. These methods have been successful in resolving several varieties of nonlinear differential equations and may offer fresh perspectives on how the problem behaves in various scenarios. The examination of the physical implications of the discovered solutions represents another area for future investigation. We may gain a deeper knowledge of the behavior of the equation and its applicability to realworld situations by investigating the qualities and characteristics of the soliton solutions, such as their stability, interaction behavior, and influence on fluid dynamics. Additionally, investigating the relationship between the discovered solutions and experimental or observational data might aid in validating the solutions' applicability for use in real-world situations.

Future studies may combine analytical methods, numerical simulations, and experimental validations to fill up these research gaps. Theoretical research might concentrate on creating novel methods to solving problems or improving already-existing ones to produce a wider range of exact solutions. To examine the equation's dynamics and confirm the stability and applicability of the discovered solutions, numerical simulations can be used. The outcomes of experimental investigations, such as laboratory tests or field measurements, can be compared to and validated with useful data. Overall, more investigation is required to examine different methods to solving the problems, examine the physical effects of the discovered solutions, and test the results using numerical simulations and experimental experiments. We are able to better understand the proposed models and their uses in fluid mechanics by filling in these research gaps, which will open the door for more thorough and accurate models in this area.

The structure of this research paper is as follows: An introduction to Sect. 1 is provided at the outset. Sect. 2 has a description of the EEM. Different structures of the soliton solutions of the YTSF model, the (2+1) dimensional NLEE, and the propagation of water waves with surface tension are described in Sect. 3. In Sect. 4 the results are described with the use of graphs. Sect. 5 contains the conclusion..

2. Materials and Methods

In this section, we'll use the YTSF model, (2+1)-dimensional NLEE, and water wave propagation with surface tension to put the above-discussed methodology into work.

2.1 Description of the exp(-ϕ(η))-expansion method

Consider the general NLPDE.

$$P(u,\ u_t,\ u_x,\ u_y,\ u_{tt},\ u_{xt},\ u_{xy},\ u_{xx},\dots)\ =\ 0. \tag{1}$$

The wave transformation for (3+1)-dimensional NLPDE is given as

$$u(x,y,z,t) = u(\eta),\qquad \eta\ =\ ax + by + cz - kt. \tag{2}$$

The wave transformation for (2+1)-dimensional NLPDE is given as

$$u(x,y,t) = u(\eta),\qquad \eta\ =\ ax + by - kt. \tag{3}$$

The wave transformation for (1+1)-dimensional NLPDE is given as

$$u(x,t) = u(\eta),\ \eta\ =\ ax - kt. \tag{4}$$

By applying wave transformation, we have

$$Q(u, u', u'', u''' \dots) = 0, \tag{5}$$

where $'$ represents derivative with respect to η.

The solutions of Eq. (5) can be expressed with the help of the EEM as

$$u(\eta) = \sum_{n=0}^{M} a_n\,(\exp(-\phi(\eta)))^n, \tag{6}$$

where a_n are constants, $a_n\ \neq\ 0$ and $0 \leq n \leq M$.

$$= \mu\exp(\phi(\eta\,))\ +\ \exp(-\phi(\eta))\ +\ \lambda,))^n, \tag{7}$$

where ' represents derivative with respect to η. Eq. (7) has the following solutions:

Family-i:

If $\mu \neq\ 0$ and $\lambda^2\ -\ 4\mu\ >\ 0$, then

$$\phi(\eta)\ = \ln(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\eta + H)\right) - \frac{\lambda}{2\mu}. \tag{8}$$

Family-ii:

If $\mu \neq\ 0$ and $\lambda^2\ -\ 4\mu\ <\ 0$, then

$$\phi(\eta)\ = \ln(\frac{\sqrt{\lambda^2 - 4\mu}}{2\mu}\tan\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\eta + H)\right) - \frac{\lambda}{2\mu}. \tag{9}$$

Family-iii:

If $\mu =\ 0, \lambda \neq 0$ and $\lambda^2\ -\ 4\mu\ >\ 0$, then

$$\phi(\eta) = -\ln\left(\frac{\lambda}{\exp(\lambda(\eta+H))-1}\right). \tag{10}$$

Family-iv:

If $\mu \neq\ 0, \lambda \neq 0$ and $\lambda^2\ -\ 4\mu =\ 0$, then

$$\phi(\eta)\ = \ln(\frac{2(\lambda(\eta+H)+2}{\lambda^2(\eta+H)}). \tag{11}$$

Family-v:

If $\mu\ =\ 0, \lambda\ =\ 0$ and $\lambda^2\ -\ 4\mu =\ 0$, then

$$\phi(\eta)\ =\ \ln(\eta\ +\ H). \tag{12}$$

2.2 Yu-Toda-Sasa-Fukuyama model

Considering the (3 + 1) dimensional YTSF model [38].

$$u_{xxxz} - 4u_{xt} + 4u_x u_{xz} + 2u_{xx}u_z + 3u_{yy} = 0, \quad (13)$$

where $p: \mathbb{R}_x \times \mathbb{R}_y \times \mathbb{R}_z \to \mathbb{R}$ in a such a way that $p = u_x$. By using wave transformation of Eq. (2), which converts the NLFDEs into NLODEs.

$$a^3cu^{(4)} + 6a^2cu'u'' + 4aku'' + 3b^2u'' = 0. \quad (14)$$

By integrating Eq. (21) with respect to η we have

$$a^3cu^{(3)} + 3a^2c(u')^2 + 4aku' + 3b^2u' = 0. \quad (15)$$

By using the balance technique on Eq. (15) between $(u')^2$ and $u^{(3)}$, we get n=1. Now for n=1, we have

$$u(\eta) = A_o + A_1 \exp\big(-\phi(\eta)\big). \quad (16)$$

where A_o and A_1 are constants to be determined. Adding Eq. (7), Eq. (8), and Eq. (16) into Eq. (15), we have

$$[C_{-4}e^{-4\phi(\eta)} + C_{-3}e^{-3\phi(\eta)} + C_{-2}e^{-2\phi(\eta)} + C_{-1}e^{-\phi(\eta)} + C_o] = 0, \quad (17)$$

where

$$C_{-4} = 3a^2{A_1}^2c - 6a^3A_1c,$$
$$C_{-3} = 6a^2{A_1}^2c\lambda - 12a^3A_1c\lambda,$$
$$C_{-2} = -7a^3A_1c\lambda^2 - 8a^3A_1c\mu + 3a^2{A_1}^2c\lambda^2 + 6a^2{A_1}^2c\mu - 4aA_1k - 3A_1b^2,$$
$$C_{-1} = a^3A_1(-c)\lambda^3 - 8a^3A_1c\lambda\mu + 6a^2{A_1}^2c\lambda\mu - 4aA_1k\lambda - 3A_1b^2\lambda,$$
$$C_0 = -a^3A_1c\lambda^2\mu - 2a^3A_1c\mu^2 + 3a^2{A_1}^2c\mu^2 - 4aA_1k\mu - 3A_1b^2\mu.$$
$$\{C_{-4} = 0, C_{-3} = 0, C_{-2} = 0, C_{-1} = 0, C_0 = 0. \quad (18)$$

Solving the system, we obtain

$$\left\{A_1 = 2a,\ k = -\frac{a^3c\lambda^2 - 4a^3c\mu + 3b^2}{4a}\right.. \quad (19)$$

Eq. (13) has the following solutions:

Family-i:

$$u_1(x, y, z, t) = \frac{2a}{-\frac{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\left(\frac{t(a^3c\lambda^2 - 4a^3c\mu + 3b^2)}{4a} + aX + bY + cZ + H\right)\right)}{2\mu} - \frac{\lambda}{2\mu}} + A_o.$$

Family-ii:

$$u_2(x,y,z,t) = \frac{2a}{-\frac{\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\left(\frac{t(a^3c\lambda^2-4a^3c\mu+3b^2)}{4a}+aX+bY+cZ+H\right)\right)}{2\mu}-\frac{\lambda}{2\mu}}+A_o.$$

Family-iii:

$$u_3(x,y,z,t) = \frac{2a\lambda}{\exp\left(\lambda\left(\frac{t(a^3c\lambda^2-4a^3c\mu+3b^2)}{4a}+aX+bY+cZ+H\right)\right)-1}+A_o.$$

Family-iv

$$u_4(x,y,z,t) = \frac{4a\left(\lambda\left(\frac{t(a^3c\lambda^2-4a^3c\mu+3b^2)}{4a}+aX+bY+cZ+H\right)+2\right)}{\lambda^2\left(\frac{t(a^3c\lambda^2-4a^3c\mu+3b^2)}{4a}+aX+bY+cZ+H\right)}+A_o.$$

Family-v

$$u_5(x,y,z,t) = \frac{2a}{\frac{3b^2t}{4a}+aX+bY+cZ+H}+A_o.$$

2.3 (2+1)-dimensional nonlinear evolution equation

Taing the (2+1)-dimensional NLEE [65].

$$u_{xxxx} + c_1u_{tt} + c_2u_{xt} + c_3u_{xy} + c_4{u^2}_{xx} = 0, \tag{20}$$

where $p: \mathbb{R}_x \times \mathbb{R}_y \to \mathbb{R}$ in a such a way that $p = u_x$ and c_1, c_2, c_3, c_4 are arbitrary constants. By using wave transformation of Eq. (3), which converts the NLFDEs into NLODEs

$$a^4 u^{(4)} + c_4(2a^2u\,u'' + 2a^2(u')^2) + abc_3 - ac_2ku'' + c_1k^2u'' = 0. \tag{21}$$

By integrating Eq. (21) with respect to η , we have

$$a^4u'' + a^2c_4u^2 + abc_3u - ac_2ku + c_1k^2u = 0. \tag{22}$$

By using the balance technique on Eq. (22) between u^2 and u'', we get n =2.
Now for $n = 2$, we have

$$u(\eta) = A_o + A_1\exp(-\phi(\eta)) + A_2\exp(-2\phi(\eta)). \tag{23}$$

Substituting Eq. (7), Eq. (8), and Eq. (23) into Eq. (22), we have

$$[D_{-4}e^{-4\phi(\eta)} + D_{-3}e^{-3\phi(\eta)} + D_{-2}e^{-2\phi(\eta)} + D_{-1}e^{-\phi(\eta)} + D_0] = 0, \tag{24}$$

where

$$D_{-4} = 6\ a^4A_2 + a^2A_2^2\, c_4,$$

$$D_{-3} = 10a^4A_2\lambda + 2a^4A_1 + + 2a^4A_1\, A_2c_4,$$

$$D_{-2} = 4a^4A_2\lambda^2 + 3a^4A_1\,\lambda + 8a^4\, A_2\mu + 2a^2\, A_2c_4A_0 + a^2A_1^2\, c_4 + aA_2\, bc_3 - aA_2\, c_2k + A_2\, c_1k^2,$$

$$D_{-1} = a^4A_1\lambda^2 + 6a^4A_2\lambda\mu + 2a^4A_1\mu + 2a^2A_1c_4A_o + aA_1bc_3 - aA_1c_2k + A_1c_1k^2,$$

$$D_0 = a^4A_1\lambda\mu + 2a^4A_2\mu^2 + a^2\, c_4A_0^2 + abc_4A_0 - ac_4\, kA_0 + c_1k^2A_0.$$

$$\begin{cases} D_{-4} = 0, D_{-3} = 0, \\ D_{-2} = 0, D_{-1} = 0, D_0 = 0. \end{cases} \tag{25}$$

Case 1:

$$\begin{cases} A_0 = -\frac{6a^2\mu}{c_4}, \\ A_0 = -\frac{6a^2}{c_4}, \\ \lambda = -\frac{A_1c_4}{6a^2}A_1^2, \\ k = \frac{\sqrt{144c_1\,(144a^4\mu - 36abc_3 + A_1^2c_2^2) + 1296a^2\,c_2^2} + 36ac_2}{72c_1}, \end{cases} \tag{26}$$

where A_0 and A_2 are free parameters. Eq. (20) has the following solutions. By using Eq. (7), Eq. (23), Eq. (24), Eq. (25), and Eq. (26), we have the following solutions:

Family-i:

$u_1(x, y, t) =$

$$-\frac{6a^2}{c_4\left(\frac{A_1c_4}{12a^2\mu} - \frac{\sqrt{\frac{A_1^2c_4^2}{36a^2} - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\frac{A_1^2c_4^2}{36a^2} - 4\mu}\left(-\frac{t(\sqrt{144c_1\left(144a^4\mu - 36abc_3 - A_1^2c_4^2\right) + 1296a^2c_2^2} + 36ac_2}{72c_1}aX + bY + H\right)\right)}{2\mu}\right)^2} + \frac{A_1}{\frac{A_1c_4}{12a^2\mu} - \frac{\sqrt{\frac{A_1^2c_4^2}{36a^2} - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\frac{A_1^2c_4^2}{36a^2} - 4\mu}\left(-\frac{t(\sqrt{144c_1\left(144a^4\mu - 36abc_3 - A_1^2c_4^2\right) + 1296a^2c_2^2} + 36ac_2}{72c_1}aX + bY + H\right)\right)}{2\mu}}$$

Family-ii:

$$u_2(x,y,t) =$$

$$-\frac{6a^2}{c_4\left(\frac{A_1c_4}{12a^2\mu}-\frac{\sqrt{4\mu-\frac{{A_1}^2{c_4}^2}{36a^2}}\tan\left(\frac{1}{2}\sqrt{4\mu-\frac{{A_1}^2{c_4}^2}{36a^2}}\left(-\frac{t(\sqrt{144c_1\left(144a^4\mu-36abc_3-{A_1}^2{c_4}^2\right)+1296a^2{c_2}^2}+36ac_2}{72c_1}aX+bY+H\right)\right)}{2\mu}\right)^2}+$$

$$\frac{A_1}{\frac{A_1c_4}{12a^2\mu}-\frac{\sqrt{4\mu-\frac{{A_1}^2{c_4}^2}{36a^2}}\tan\left(\frac{1}{2}\sqrt{4\mu-\frac{{A_1}^2{c_4}^2}{36a^2}}\left(-\frac{t(\sqrt{144c_1\left(144a^4\mu-36abc_3-{A_1}^2{c_4}^2\right)+1296a^2{c_2}^2}+36ac_2}{72c_1}aX+bY+H\right)\right)}{2\mu}}$$

Family-iii:

$$u_3(x,y,t) = -\frac{6a^2\mu}{c_4}-$$

$$\frac{{A_1}^2c_4}{6a^2\left(\exp\left(-\frac{\left(-\frac{A_1c_4\,(-t(\sqrt{144c_1\left(144a^4\mu-36abc_3-{A_1}^2{c_4}^2\right)+1296a^2{c_2}^2}+36ac_2)}{72c_1}aX+bY+H\right)}{2\mu}\right)-1\right)}-$$

$$\frac{{A_1}^2c_4}{6a^2(\exp\left(-\frac{\left(-\frac{A_1c_4\,(-t(\sqrt{144c_1\left(144a^4\mu-36abc_3-{A_1}^2{c_4}^2\right)+1296a^2{c_2}^2}+36ac_2)}{72c_1}aX+bY+H\right)}{2\mu}\right)-1)^2}.$$

Family-iv:

$$u_4(x,y,t) = -\frac{6a^2\mu}{c_4}+\frac{72a^4(\frac{\left(2-\frac{A_1c_4\,(-t(\sqrt{144c_1\left(144a^4\mu-36abc_3-{A_1}^2{c_4}^2\right)+1296a^2{c_2}^2}+36ac_2)}{72c_1}aX+bY+H\right)}{6a^2})}{A_1{c_4}^2\left(2-\frac{A_1c_4\,(-t(\sqrt{144c_1\left(144a^4\mu-36abc_3-{A_1}^2{c_4}^2\right)+1296a^2{c_2}^2}+36ac_2)}{72c_1}aX+bY+H\right)}-$$

$$\frac{3110a^{10}(\frac{\left(2-\frac{A_1c_4\,(-t(\sqrt{144c_1\left(144a^4\mu-36abc_3-{A_1}^2{c_4}^2\right)+1296a^2{c_2}^2}+36ac_2)}{72c_1}aX+bY+H\right)^2}{6a^2})}{{A_1}^4{c_4}^5\left(\frac{-t(\sqrt{144c_1\left(144a^4\mu-36abc_3-{A_1}^2{c_4}^2\right)+1296a^2{c_2}^2}+36ac_2)}{72c_1}aX+bY+H\right)^2}$$

Family-v:

$$u_4(x,y,t) = -\frac{6a^2}{c_4\left(\frac{-t(\sqrt{144c_1(-36abc_3-A_1{}^2c_4{}^2)+1296a^2c_2{}^2}+36ac_2)}{72c_1}aX+bY+H\right)^2} + \frac{A_1}{\frac{-t(\sqrt{144c_1(-36abc_3-A_1{}^2c_4{}^2)+1296a^2c_2{}^2}+36ac_2)}{72c_1}aX+bY+H} + A_0.$$

2.4 Water wave propagation with surface tension

Finally, taking equation of water wave propagation with surface tension [60].

$$u_{tt} - u_{xx} + au_{xxxx} - bu_{xxtt} + u_t u_{xx} + 2u_x u_{xt} = 0, \quad (27)$$

where a and b are arbitrary constants. With the help of wave transformation of Eq. (4), we have

$$ak^4u^{(4)} - bc^2k^2u^{(4)} + c^2u'' - 3ck^2u'u'' - k^2u'' = 0. \quad (28)$$

By integrating Eq. (28) with respect to , we have

$$ak^4u^{(4)} - bc^2k^2u^{(3)} + c^2u' - \frac{3}{2}ck^2 (u')^2 - k^2u' = 0. \quad (29)$$

By using the balance technique on Eq. (29) between $(u')^2$ and $u^{(3)}$, we get $n = 1$. Now for $n = 1$, we have

$$u(\eta) = A_o + A_1\, exp(-\Phi(\eta)). \quad (30)$$

Substituting Eq. (7), Eq. (8), Eq. (30) into Eq. (27), we have

$$[E_{-4}e^{-4\Phi(\eta)} + E_{-3}e^{-3\Phi(\eta)} + E_{-2}e^{-2\Phi(\eta)} + E_{-1}e^{-\Phi(\eta)} + E_0] = 0, \quad (31)$$

where

$$E_{-4} = -6aA_1k^4 + 6A_1bc^2k^2 - \frac{3}{2}A_1{}^2c\,k^2,$$

$$E_{-3} = -12aA_1\lambda\, k^4 + 12A_1bc^2\lambda\, k^2 - 3A_1{}^2c\,\lambda\, k^2,$$

$$E_{-2} = -7aA_1\lambda^2k^4 - 8aA_1\, k^4\mu + 7A_1bc^2\lambda^2k^2 + 8A_1bc^2k^2\mu + A_1c^2 - \frac{3}{2}A_1{}^2c\,\lambda^2k^2 - 3A_1{}^2c\,k^2\mu + A_1\, k^2,$$

$$E_{-1} = -aA_1\lambda^3k^4 - 8aA_1\lambda\ k^4\mu + A_1bc^2\lambda^3k^2 + 8A_1bc^2\lambda k^2\mu - A_1c^2\lambda - 3A_1{}^2c\lambda k^2\mu + A_1\lambda k^2,$$

$$E_0 = -aA_1\lambda^2k^4\mu - 2aA_1\ k^4\mu^2 + A_1bc^2\lambda^2k^2\mu + 2A_1bc^2k^2\mu^2 - A_1c^2\mu - \frac{3}{2}A_1{}^2c\ k^2\mu^2 + A_1\ k^2\mu,$$

$$\begin{cases} E_{-4} = 0, E_{-3} = 0, \\ E_{-2} = 0, E_{-1} = 0, E_0 = 0. \end{cases} \tag{32}$$

Solving the system, we obtain

Case1:

$$\begin{cases} A_1 = -\dfrac{\frac{2abc^2\lambda^2}{a\lambda^2-4a\mu} - \frac{8abc^2\mu}{a\lambda^2-4a\mu} - \frac{2a\sqrt{(-bc^2\lambda^2+4bc^2\mu-1)^2-4c^2(a\lambda^2-4a\mu)}}{a\lambda^2-4a\mu} + \frac{2a}{a\lambda^2-4a\mu} - 4bc^2}{c}, \\ K = -\sqrt{-\frac{2bc^2\mu}{a\lambda^2-4a\mu} + \frac{bc^2\lambda^2}{2(a\lambda^2-4a\mu)} - \frac{\sqrt{(-bc^2\lambda^2+4bc^2\mu-1)^2-4c^2(a\lambda^2-4a\mu)}}{2(a\lambda^2-4a\mu)} + \frac{1}{2(a\lambda^2-4a\mu)}}. \end{cases} \tag{33}$$

By using Eq. (6), Eq. (7), Eq. (30), Eq. (32) and Eq. (33), we have the following solutions:

Family i:

$$u_1(x,t) = Ao - \frac{\frac{2abc^2\lambda^2}{a\lambda^2-4a\mu} - \frac{8abc^2\mu}{a\lambda^2-4a\mu} - \frac{2a\sqrt{(-bc^2\lambda^2+4bc^2\mu-1)^2-4c^2(a\lambda^2-4a\mu)}}{a\lambda^2-4a\mu} + \frac{2a}{a\lambda^2-4a\mu} - 4bc^2}{c\left(-\frac{\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}\left(-X\sqrt{-\frac{2bc^2\mu}{a\lambda^2-4a\mu}+\frac{bc^2\lambda^2}{2(a\lambda^2-4a\mu)}-\frac{\sqrt{(-bc^2\lambda^2+4bc^2\mu-1)^2-4c^2(a\lambda^2-4a\mu)}}{2(a\lambda^2-4a\mu)}+\frac{1}{2(a\lambda^2-4a\mu)}}-ct+H\right)\right)}{2\mu} - \frac{\lambda}{2\mu}\right)}$$

Familyii:

$$u_2(x,t) = Ao - \frac{\frac{2abc^2\lambda^2}{a\lambda^2-4a\mu} - \frac{8abc^2\mu}{a\lambda^2-4a\mu} - \frac{2a\sqrt{(-bc^2\lambda^2+4bc^2\mu-1)^2-4c^2(a\lambda^2-4a\mu)}}{a\lambda^2-4a\mu} + \frac{2a}{a\lambda^2-4a\mu} - 4bc^2}{c\left(\frac{\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}\sqrt{4\mu-\lambda^2}\left(-X\sqrt{-\frac{2bc^2\mu}{a\lambda^2-4a\mu}+\frac{bc^2\lambda^2}{2(a\lambda^2-4a\mu)}-\frac{\sqrt{(-bc^2\lambda^2+4bc^2\mu-1)^2-4c^2(a\lambda^2-4a\mu)}}{2(a\lambda^2-4a\mu)}+\frac{1}{2(a\lambda^2-4a\mu)}}-ct+H\right)\right)}{2\mu} - \frac{\lambda}{2\mu}\right)}$$

Familyiii:

$$u_3(x,t) = Ao - \frac{\frac{2abc^2\lambda^2}{a\lambda^2-4a\mu} - \frac{8abc^2\mu}{a\lambda^2-4a\mu} - \frac{2a\sqrt{(-bc^2\lambda^2+4bc^2\mu-1)^2-4c^2(a\lambda^2-4a\mu)}}{a\lambda^2-4a\mu} + \frac{2a}{a\lambda^2-4a\mu} - 4bc^2}{c\left(exp\left(\lambda\left(-X\sqrt{-\frac{2bc^2\mu}{a\lambda^2-4a\mu}+\frac{bc^2\lambda^2}{2(a\lambda^2-4a\mu)}-\frac{\sqrt{(-bc^2\lambda^2+4bc^2\mu-1)^2-4c^2(a\lambda^2-4a\mu)}}{2(a\lambda^2-4a\mu)}+\frac{1}{2(a\lambda^2-4a\mu)}}-ct+H\right)\right)-1\right)}$$

Familyiv:

$$u_4(x,t) = 4\left(\lambda x\sqrt{\frac{-2\sqrt{(bc^2(\lambda^2-4\mu)+1)^2-4ac^2(\lambda^2-4\mu)}+2bc^2(\lambda^2-4\mu)+2}{a(\lambda^2-4\mu)}}+2c\lambda t-2H\lambda-4\right)\left(\frac{\sqrt{(bc^2(\lambda^2-4\mu)+1)^2-4ac^2(\lambda^2-4\mu)}+bc^2(\lambda^2-4\mu)-1}{c\lambda^2(\lambda^2-4\mu)\left(X\sqrt{\frac{-2\sqrt{(bc^2(\lambda^2-4\mu)+1)^2-4ac^2(\lambda^2-4\mu)}+2bc^2(\lambda^2-4\mu)+2}{a(\lambda^2-4\mu)}}+2ct-2H\right)}\right)+Ao.$$

Familyiv:

$$u_5(x,t) = Ao + \frac{4bc}{-ct+H+kx}.$$

3. Results and Discussion

In this section, we provide a graphical representation of our proposed method and demonstrate its validity by numerical simulation of all sections of models. Solitons are physically represented by localized wave packets or envelopes in the NLEEs that maintain their form and move across the medium without distortion or dispersion. The structures of their dynamical behavior are investigated using the physical interpretation of the given solutions. This is demonstrated by numerical simulations, which are accomplished by selecting appropriate arbitrary functions as well as constants appearing in equation solutions. We realize that the solutions discovered may be helpful in comprehending different physical phenomena. The needed research is successful in coming up with novel solutions to the necessary models. Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, and 15 display the acquired solutions in three-dimensional (3D), two-dimensional (2D), and contour graphs to describe the physical behavior of waves propagating in a nonlinear medium. There are many different parameters in these solutions. Because the parameters determine the shape of the solution, we can generate a wide range of graphs by changing their values. They include solutions for periodic, dark, bright, hyperbolic, kink, and hyperbolic soliton systems. They are extremely useful for regulating information transfer in optical fiber networks and controlling the dynamics of light pulses. They have the ability to convey information, behave like particles, and might be used in spintronics, quantum computing, and energy-efficient electronics. These recently discovered exact solutions have significant physical consequences.

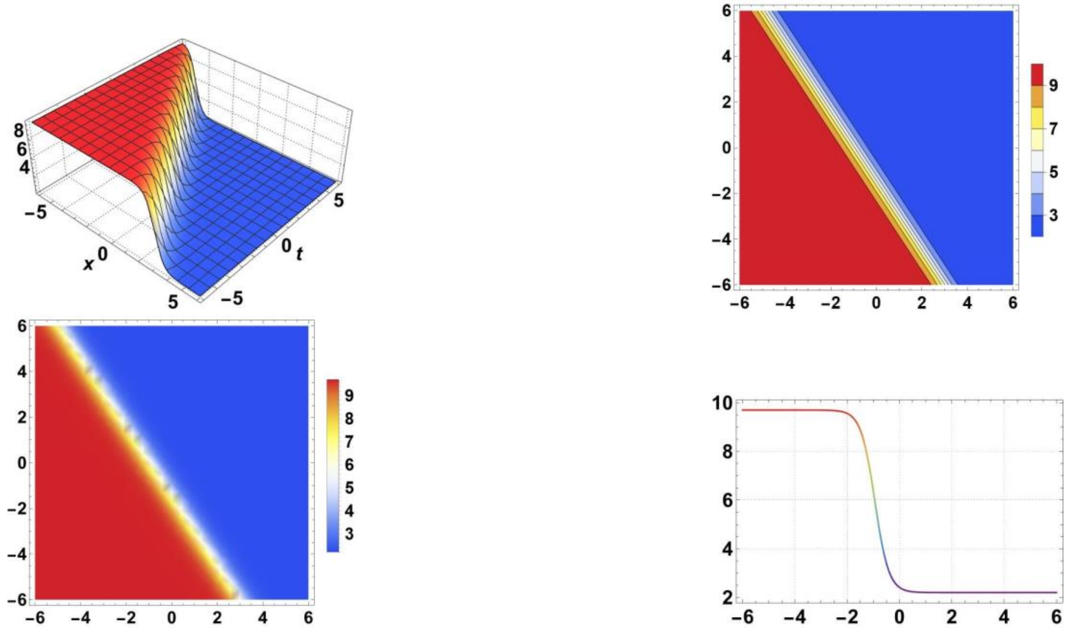

Fig. 1. *3D, contour, density, and 2D plots via u_1 (x,y,z,t) with μ = 1,z = 1.2,a = 2.5,b = 0.9,λ = 2.5,c = 0.4,A_o= 0.3,H = 0.5, and y = 0.5, for - 6 ≤ x ≤ 6,-6 ≤ t ≤ 6.*

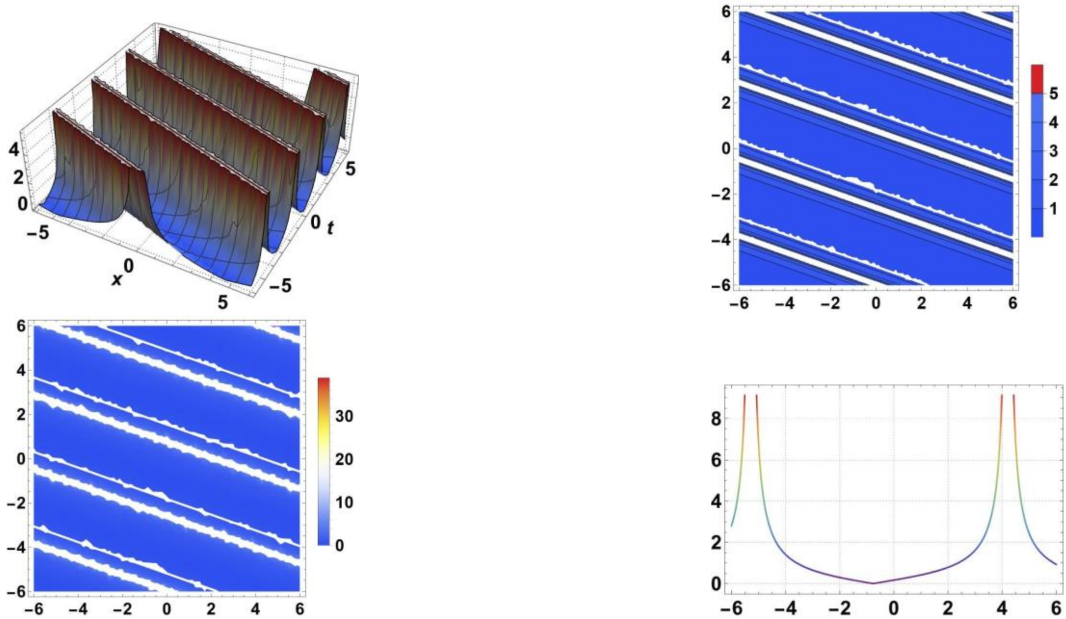

Fig. 2. 3D, contour, density, and 2D plots via u_1 (x,y,z,t) with μ = 1,z = 1.2,a = 2.5,b = 0.9,λ = 2.5,c = 0.4,A_o= 0.3,H = 0.5, and y = 0.5, for - 6 ≤ x ≤ 6,-6 ≤ t ≤ 6

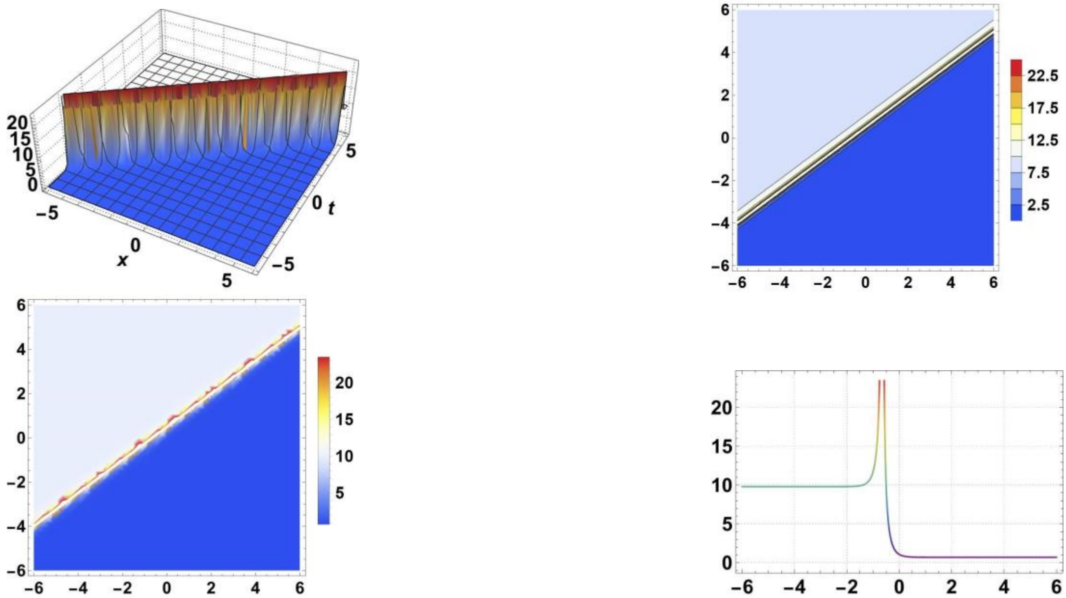

Fig. 3. **3D, contour, density, and 2D plots via u_3 (x,y,z,t) with $\mu = 1$, $c = 0.9, a = 3.5, H = 1.5, b = 1.8, \lambda = 1.5, b = 0.8, A\_o = 0.7, y = 0.5$, and $z = 0.4$, for $-6 \le x \le 6, -6 \le t \le 6$.**

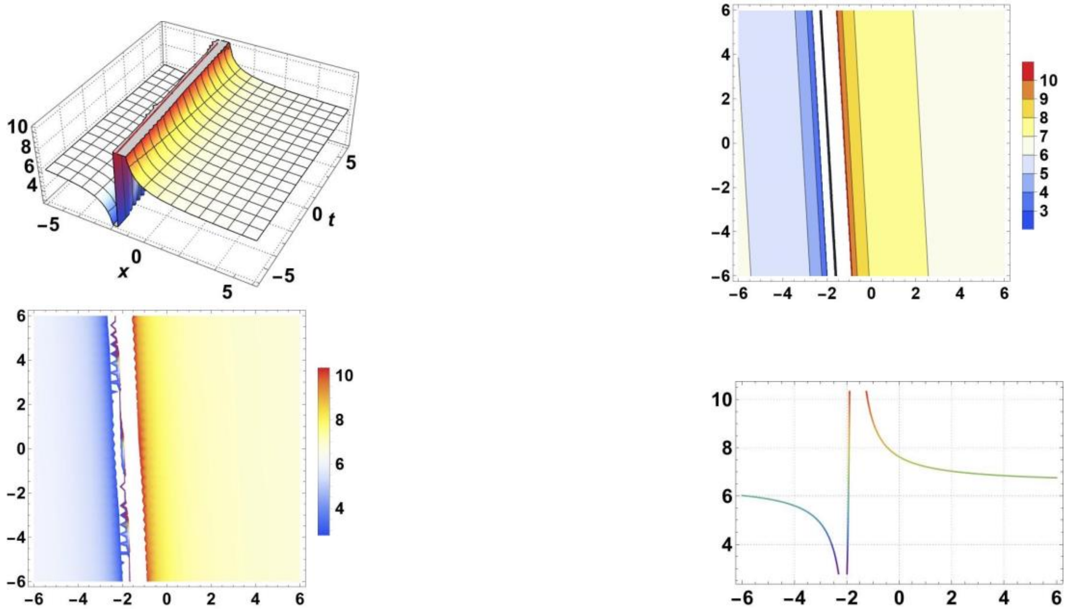

Fig. 4. **3D, contour, density, and 2D plots via u_4 (x,y,z,t) with $\mu = 1, c = 0.5, a = 2.9, H = 1.5, b = 1.6, \lambda = 2, b = 0.8$, ⟦ A⟧_o $= 0.7, y = 2.7$, and $z = 3$, for $-6 \le x \le 6, -6 \le t \le 6$.**

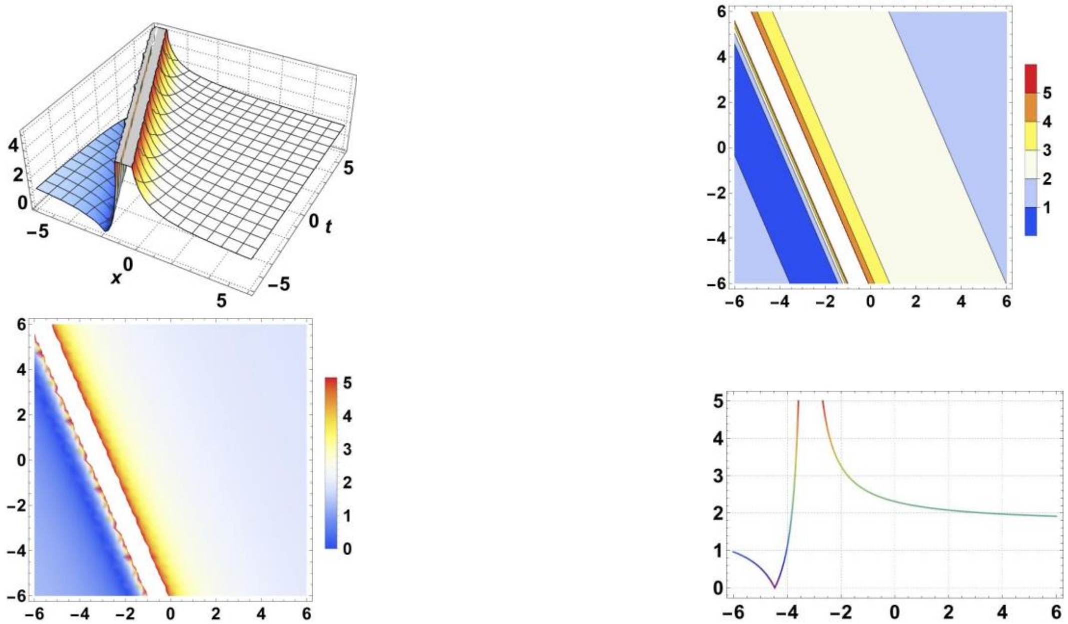

***Fig. 5*. 3D, contour, density, and 2D plots via u_5 (x,y,z,t) with c = 1.7,a = 2.9,H = 1.5,b = 2.2,A_o= 1.7,y = 2.1,and z = 2, for - 6 ≤ x ≤ 6,-6 ≤ t ≤ 6.**

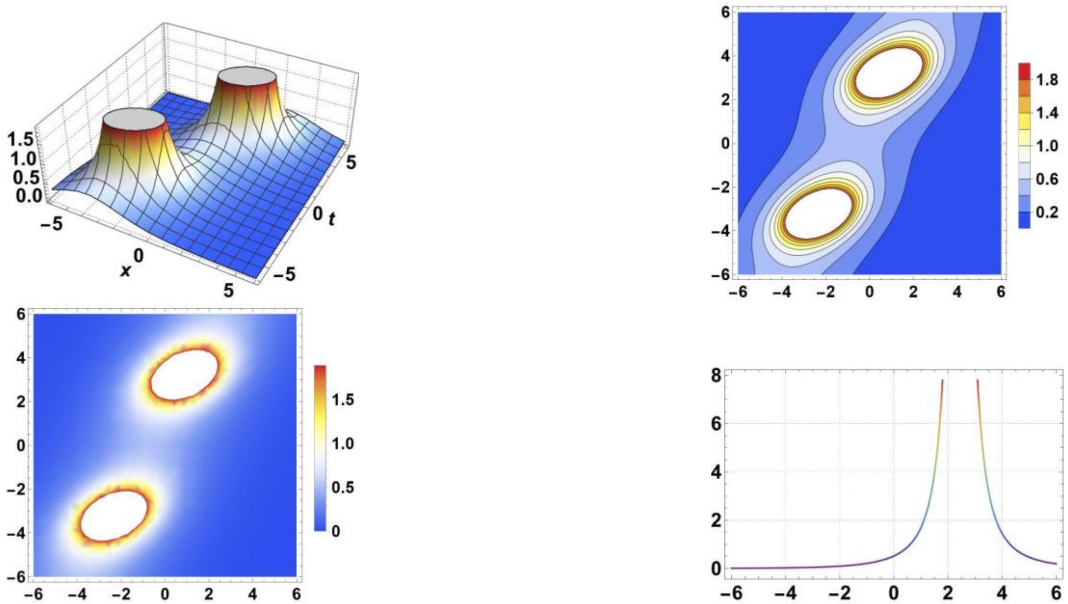

***Fig. 6*. 3D, contour, density, and 2D plots via u_1(x, y, t) with μ = 0.5,〖 c〗_4 = 1.5,a = 0.3,A_1 = 1,H = 0.8,y = 1,〖 c〗_2 = 0.8,〖 c〗_1 = 0.8,〖 c〗_3 = 0.7,and b = 0.5,for - 6 ≤ x ≤ 6,-6 ≤ t ≤ 6.**

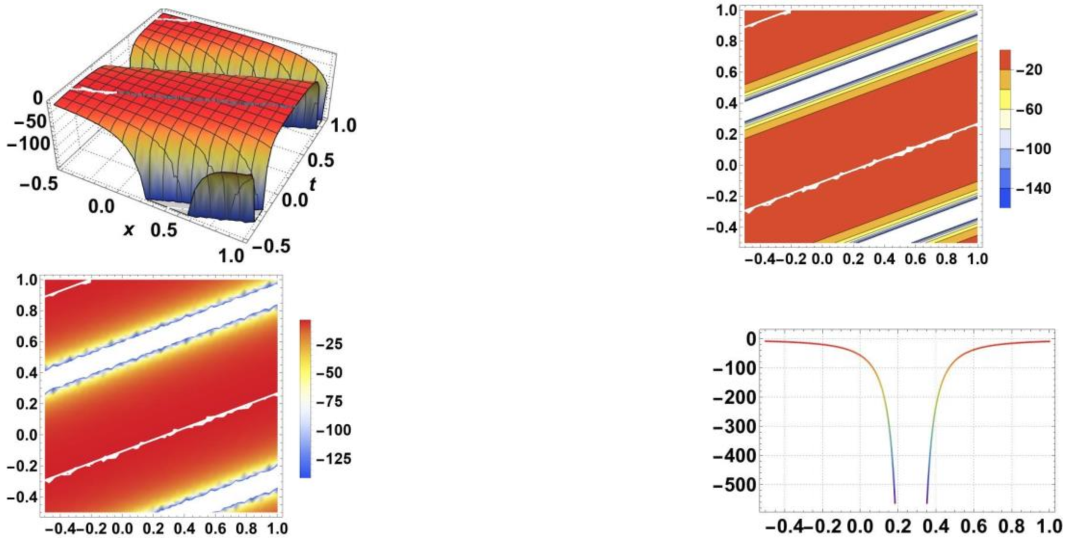

Fig. 7. **3D, contour, density, and 2D plots via u_2 (x,y,t) with μ = 1,〚 c〛_4 = 1.5,a = 1,〚 A〛_1 = 1,H = 0.8, y = 1,〚 c〛_2 = 0.8,〚 c〛_1 = 0.8,〚 c〛_3 = 0.7,and b = 0.5,0 - 0.5 ≤ x ≤ 1,-0.5 ≤ t ≤ 1.**

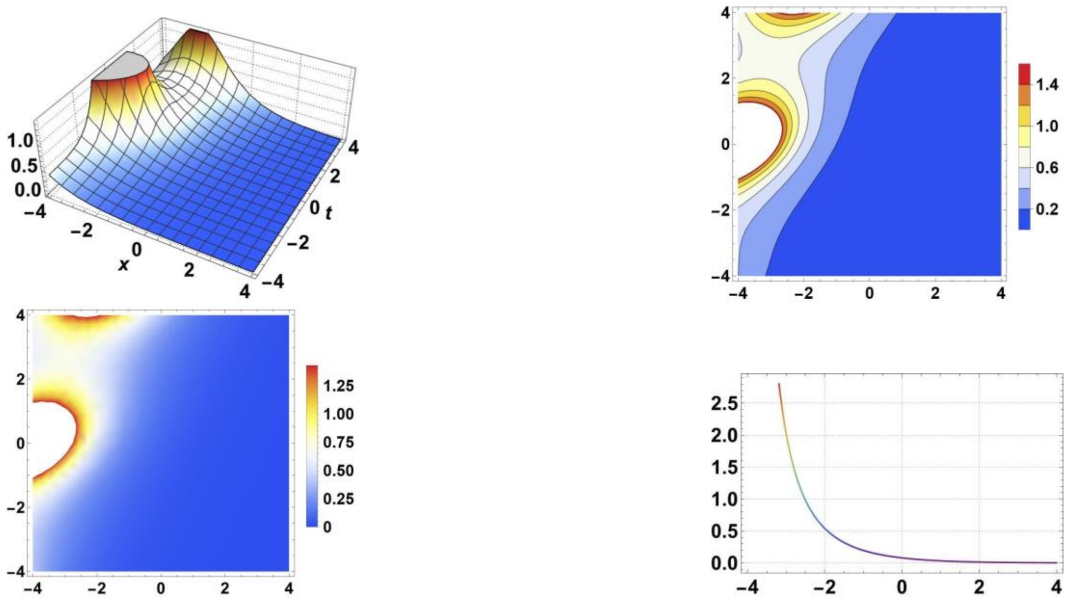

Fig. 8. **3D, contour, density, and 2D plots via u_3 (x,y,t) with μ = 0,〚 c〛_4 = 1.5,a = 0.3,A_1 = 0.8,y = 1,〚 c〛_2 =0.8,c_1 = 0.8,〚c 〛_3= 0.7,and b = 0.5,for - 4 ≤ x ≤ 4,-4 ≤ t ≤ 4.**

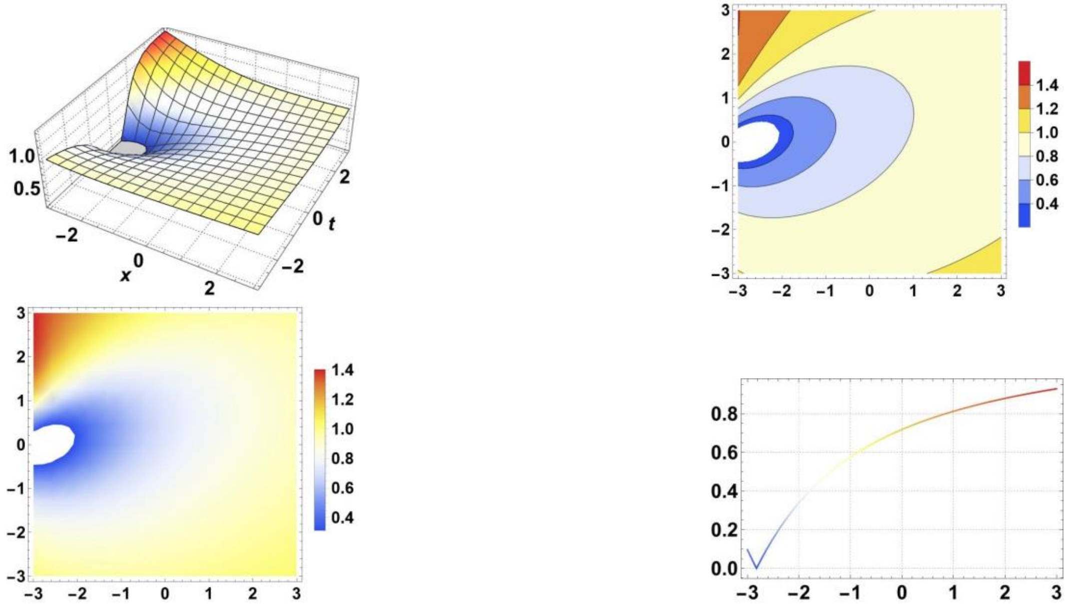

Fig. 9. **3D, contour, density, and 2D plots via $u_4(x,y,t)$ with $\mu = 1$, $c_4 = 1.5$, $a = 0.3$, $A_1 = 1$, $H = 0.8$, $y = 1$, $c_2 = 0.8$, $c_1 = 0.8$, $c_3 = 0.7$, and $b = 0.5$, for $-3 \le x \le 3$, $-3 \le t \le 3$.**

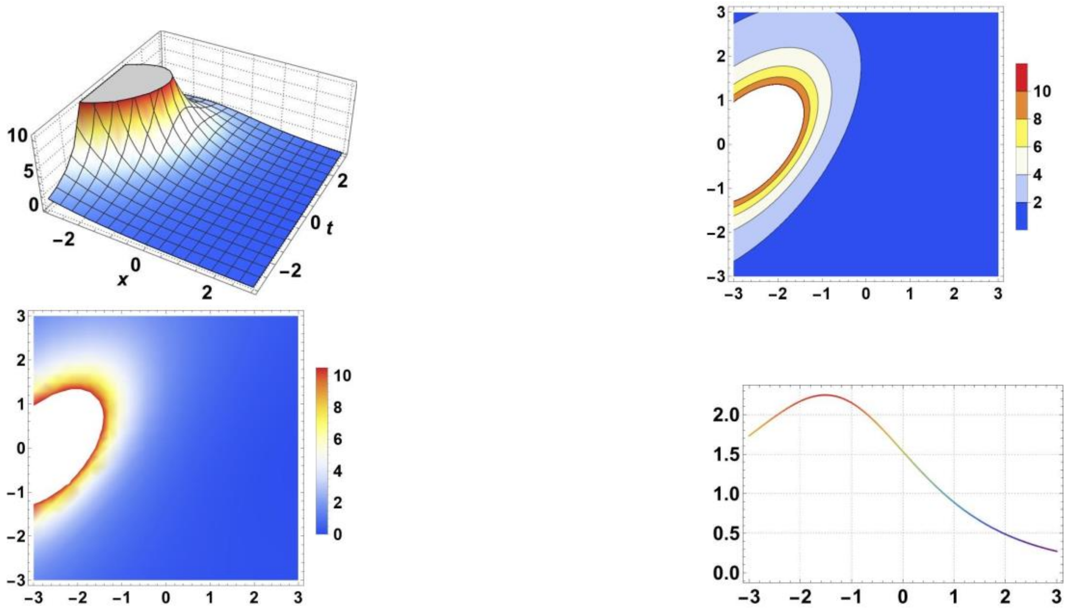

Fig. 10. **3D, contour, density, and 2D plots via $u_5(x,y,t)$ with $\mu = 0$, $c_4 = 0.5$, $a = 0.5$, $A_o = 0$, $H = 0.8$, $y = 1$, $c_2 = 0.8$, $c_1 = 0.8$, $c_3 = 0.7$, and $b = 0.5$, for $-3 \le x \le 3$, $-3 \le t \le 3$.**

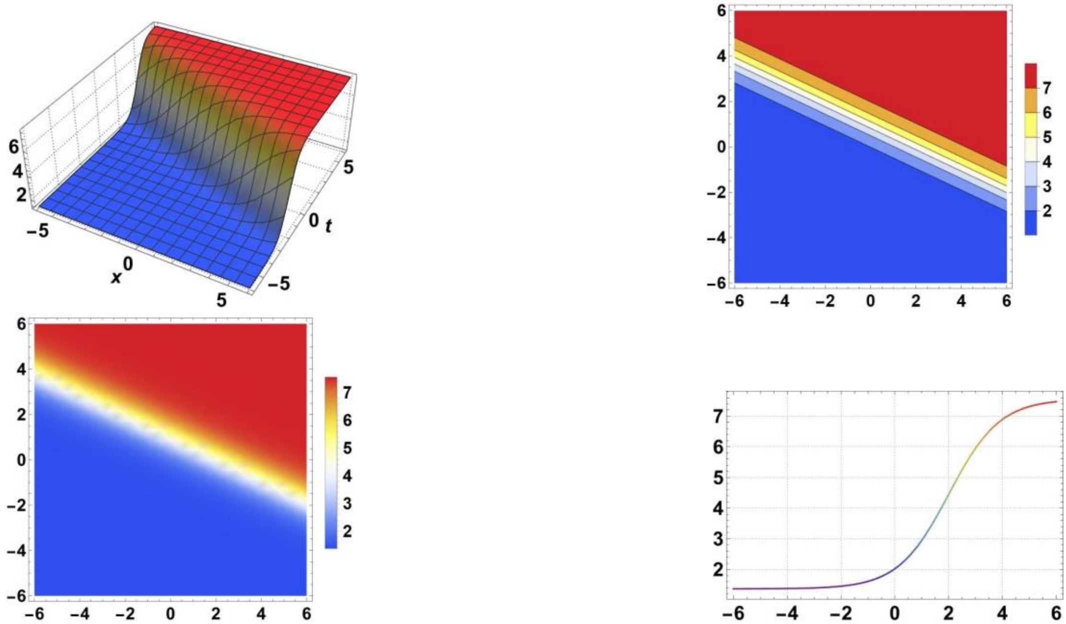

Fig. 11. **3D, contour, density, and 2D plots via 〚$u_1 (x,t)$ with $\mu = 1, c = 1.5, a = 0.5, H = 0.5, b = 1, \lambda = 2.5, b = 0.8$, and $A_o = 0.7$, for $-6 \le x \le 6, -6 \le t \le 6$.〛**

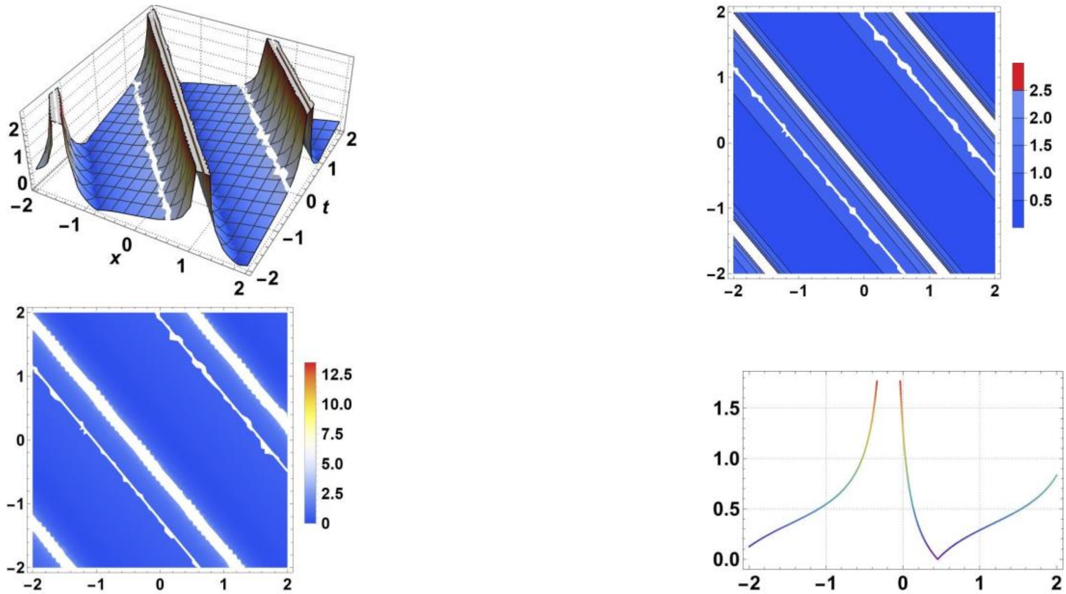

Fig. 12. 3D, contour, density, and 2D plots via $u_2 (x,t)$ with $\mu = 1, c = 1.5, a = 0.5,$ 〖 A〗$_1 = 1, H = 0.5, b = 1, \lambda = 1.5, b = 0.8$, and $A_o = 0.7$, for $-2 \le x \le 2, -2 \le t \le 2$.

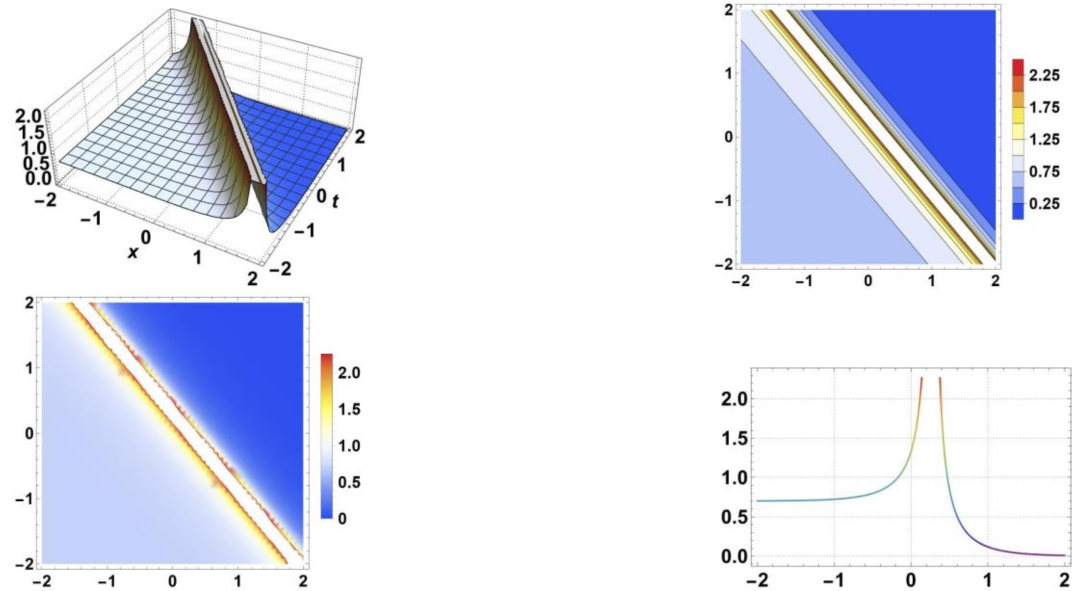

Fig. 13. 3D, contour, density, and 2D plots via u_3 (x,t)with μ = 1,c = 1.5,a = 0.5,A_1 = 1,H = 0.5,b = 1,λ = 1.5,b = 0.8,and A_o = 0.7,for - 2 ≤ x ≤ 2,-2 ≤ t ≤ 2.

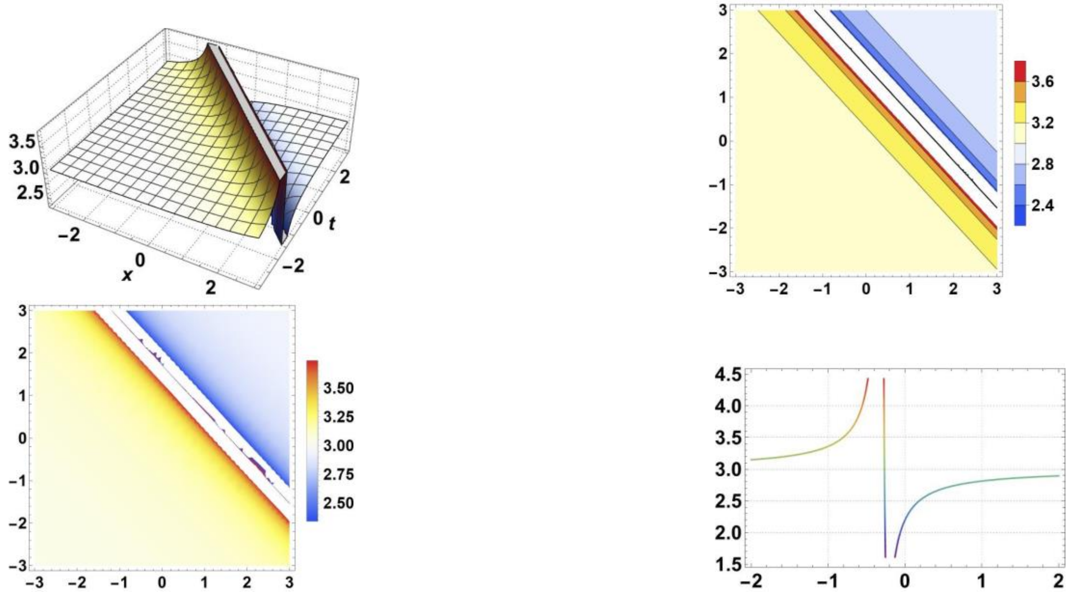

Fig. 14. 3D, contour, density, and 2D plots via u_4 (x,t) with μ = 1,c = 1.5,a = 1.5,H = 2.5,b = 2.5,λ =1.5 ,b = 1.8,and A_o = 2.7,for - 3 ≤ x ≤ 3,-3 ≤ t ≤ 3.

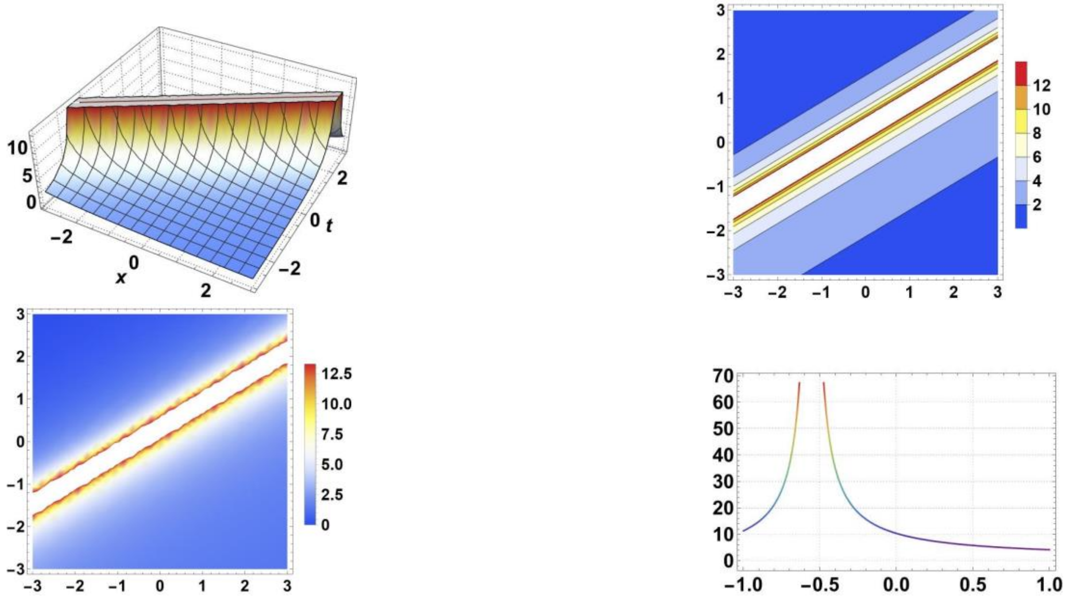

***Fig. 15.** 3D, contour, density, and 2D plots via u_5 (x,t) with c = 1.5,a = 0.5 ,A_1 = 1,H = 0.5,b = 1,b =0.8, A_o = 0.7,and k = 0.9,for - 3 ≤ x ≤ 3,-3 ≤ t ≤ 3.*

4. Conclusion

The EEM is used in this paper to investigate the aforementioned models. By applying the proposed method, we obtained several solutions in the form of hyperbolic functions and exponential function solutions. Researching NLPDEs in mathematics can be done effectively using the EEM. The exact soliton solutions are tremendous and exquisite for researchers and mathematicians due to their practical applications in engineering. Solitons may arise in water waves and are crucial for the study of rogue waves and tsunamis. Optical solitons can be visualized as localized intensity peaks or waveforms that propagate through the fiber without spreading out or deforming. In order to develop structures and coastal protection measures, solitary wave models are used to comprehend and forecast the behavior of huge waves in oceans and coastal areas. The outcomes are applicable to many academic disciplines, notably fluid dynamics. The computational effort used in addition to graphical representations enhances the proposed method's accuracy. The calculated solutions in this study are wider than in earlier studies.

The received solutions in this study span a variety of wave types, including bell-type, hyperbolic solitary wave solutions, dark, bright, periodic, kink, singular, and more. These diverse solutions have a number of advantages in several scientific and technical domains. Understanding localized wave phenomena, such as depressions and amplifications, which are important in coastal studies, wave transformation, and coastal dynamics, requires knowledge of dark and bright solitary wave solutions. In order to forecast wave propagation and interference, periodic wave solutions are useful for evaluating wave behavior across time [66]. Kink solitary wave solutions contribute to the understanding of wave-breaking events and nonlinear dynamics by explaining sudden shifts and discontinuities in wave profiles [67]. Bell-type solitary wave solutions can be used to describe localized disturbances and coherent structures, whereas singular wave solutions offer insights into severe wave occurrences and aberrant wave behavior [68]. For the study of stability and interaction phenomena in fluid dynamics, optics, and plasma physics, hyperbolic solitary wave solutions are crucial. Overall, the wide range of wave solutions discovered in this work offers insightful information on the system's complexity and nonlinear behavior, encouraging future investigation and improving our knowledge of related physical systems.

The method proposed in this study is both conventional and straightforward for dealing with challenging and time-consuming algebraic calculations. The required conclusions are recent and have never been covered in the literature. Solitons have an impact on phase changes and material characteristics. Research is now being done to determine their function in regulating and modifying the physical characteristics of materials, which might lead to improvements in the disciplines of nanotechnology and materials engineering. Studying the dynamics and interactions of solitons is crucial for understanding their stability and behavior in realistic conditions. In the future, research may concentrate on creating unique methods for regulating soliton dynamics, manipulating soliton interactions, and improving their stability. The updated generalized rational exponential function method can be used in the future to study our recommended models.

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