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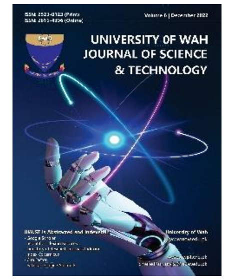


# Analysis of Wind Energy Data at Two Different Heights

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**Abstract**
The purpose of the study is to identify the best-fitted distribution(s) to provide insight into the annual maximum Wind Speed and Wind Direction. The data on wind speed in a daily time series format was gathered from the Pakistan Meteorological Department Karachi for 29 years (1990 to 2018). The measurements were recorded at 10m and 50m heights. First, we verified the fundamental assumption, such as the randomness of the data series using the Run test. For this purpose, two parameters Frechet, Log-logistic, and Weibull distributions are fitted using maximum likelihood and the Bayesian estimation methods. The performance of the candidate distributions was evaluated using the Kolmogorov Smirnove test at a 5% significance threshold. Furthermore, density functions' graphs were utilized to validate the Wind Speed data's behavior and theoretical framework. The resultant estimates depicted that for Wind Speed all three distributions are best fitted with both estimation methods but Weibull and Loglogistic distributions are found to be the most appropriate for Wind Direction with both estimation methods. From the estimated result it is clear that Weibull and Log Logistics distributions are best fitted for both Wind Speed and Wind Direction at 10 m and 50m heights.

**Keywords**
Frechet distribution; Webiull distribution; Log-logistic distribution; Maximum likelihood method; Bayesian method.

## 1. Introduction

At present, wind energy technology has reached such a level that it is being used as an important source of electricity in many countries. Since Wind is the world's most renewable and quickest energy source, it is clean and provides several benefits to humans. Also, it is non-renewable, site-dependent, and pollution-free, so serve as an important potential source of alternative energy. For the improvement of wind energy, the continued interest in around the world has led to a solid improvement in the knowledge and efficiency of power plants of wind. Energy consumption increases with technological advancement and population growth. To overcome these problems, other sources of energy like solar, geothermal and wind energy are recommended. Unlike fossil fuels, most of these energy sources that reduce carbon emissions are unlimited. As a result, these energy sources have recently attracted considerable attention. Wind power is becoming increasingly popular around the world. Pakistan's energy demand has recently surged and is expected to rise further in the future. Efficient use of energy is to change wind plants into important conformation of electricity. Distribution of wind speed is vital for power generators. Just like other energy sources, wind energy is a solar resource. The wind system is mainly designed for two main reasons.

- Temperature variations in between the equator and the poles (earth's latitudes).
- The earth's movement.

Currently, wind analysis provides valuable information to academics interested in renewable energy. The utilization of air energy reduces the combustion of fossil fuels and, as a result, carbon dioxide emissions. Because of today's energy issue and growing environmental awareness, clean energy and renewable energy sources have now gained popularity. They need to replenish their energy supply. Statistical features are required for wind energy conversion system energy prediction. Due to the great unevenness in space and time of wind energy, it is vital to demonstrate that the analytical method employed to acquire wind data would generate anticipated energy that is close to the real energy obtained. Numerous efforts have been done in recent years to build an appropriate model for the analysis of wind speed frequency distribution. During the past two decades, much emphasis has been placed on the development of improved data models for explanations of the wind speed frequency distribution. Pakistan's energy demand has recently surged and is expected to rise further rapidly which necessitates the study of wind speed energy and how to increase it. The key objective of the present work is to suggest the appropriate probability distribution(s) for Wind Speed and Wind Direction data**.**

### 1.1 Relative Studies

Numerous investigations into the distribution of wind speed have been made. For instance, [1-2] various distributions to analyze wind speed data. Moreover, [3] selected five statistical models to model the Miri data of wind speed. In comparison to the Erlag, Rayleigh, and Weibull distributions, they claimed that the lognormal and gamma distributions were significantly more suitable. Additionally, [4-5] studied the best fitted distributions. Similarly [6] evaluated average wind speed data in Bitlis for the years 2012 to 2016. The estimations were done for Lognormal, Gamma, and Weibull distributions and maximum likelihood method was applied for estimation of the parameter of the distributions. A statistical analysis was performed by [7] to identify the precise frequency distribution that fits the wind speed data. Three parameter Weibull with two parameters lognormal, Weibull, and gamma distributions studied by [8]. The maximum likelihood approach, estimates of these distributions were derived. To choose the best-fitting distribution, they employed the Chi square test, Bayesian information criteria, Kolmogorov-Smirnov test, coefficient of determination, Akaike information criterion, and root mean square error.

A study made on Wind speed and direction and predict wind energy availability for a sustainable ecosystem by [9]. Evaluation and Analysis of Wind Speed with the Weibull and Rayleigh Distribution Models for Energy Potential studied by [10]. Similarly, [11] studied for Knowledge of wind energy and [12] Reviewed on Modern Wind Power Generation Forecasting Technologies. Another studied on Strategic analysis of wind energy potential and optimal turbine selection in Saudi Arabia made by [13].

A study explored by [14] wind speed distribution for selection of the Weibull model. The scale and shape parameters were determined on hourly average data of wind speed. A study examined on wind speed data [15] using Weibull distribution to find out characteristics of wind energy conversion in Bangladesh. Solar and wind energy properties identified by [16] in Tehran. To calculate the region's solar energy potential, they utilized a method based on temperature. A Research conducted by [17] in different regions of Spain to assess the significance of numerous fitting methods. Furthermore, [18] provided a comparative analysis of some techniques for estimating the Weibull parameters for various locations.

This article introduces Wind Energy at two Different heights. The structure of the article is as follows: In section 1 introduction and relevant studies are presented. The Section 2, 3 and 4 followed by Data description, Screening of data and Candidate distributions with estimation Methods. Section 5, Section 6 and Section 7 provides useful information about Results and discussions, Analysis of wind direction data and Pdf graphs year wise. while Section 8 provides information about Comparison. Finally, the article concludes with Section 10.

## 2. Data Description and Selected Distributions

The data on wind speed in a daily time series format was collected from the Pakistan Meteorological Department Karachi for 29 years (1990 to 2018). At the Pakistan Meteorological Department Karachi, a cup generator anemometer continuously captured the Wind Speed data and this data was only available for 10m and 50m heights.

There are a large number of distributions that can be used for the Wind Speed and Wind Direction data. But in this study three two-parametric distributions namely Frechet, log-logistic and Weibull distributions are used to model the distribution of Wind Speed (WS) and Wind Direction.

## 3. Screening of Data

- **Run Test**

The run test is applied to primarily check the randomness of WS and Wind Direction data, which is

$$Z = \frac{R-\bar{R}}{S_R} \tag{1}$$

If the null hypothesis is rejected, it indicates that the data do not follow the random pattern and vice versa.

## 4. Candidate Distributions and Estimation Methods

To determine the most suitable distribution(s) for WS and Wind Direction, we consider Frechet distribution (FD), Log-logistic distribution (LLD), and Weibull distribution (WD). These distributions' details, Maximum Likelihood Estimator (MLE) and Bayesian Estimator (BE) are presented below.

- **FD**

In 1927, French mathematician Maurice Rene Frechet [19], proposed FD to evaluate extreme events. The probability density function (PDF) and cumulative distribution function (CDF) of FD are

$$f(x;\ \alpha,\beta) = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{x}\right)^{\alpha+1} exp^{-\left(\frac{\beta}{x}\right)^{\alpha}},\quad x > 0,\ \alpha,\beta > 0 \tag{2}$$

$$F(x) = exp\left[-\left(\frac{\beta}{x}\right)^{\alpha}\right],\qquad x > 0 \tag{3}$$

Where α and β are the shape and scale parameters of FD.

- **MLE for FD**

Let $x_1, x_2, x_3, \dots, x_n$ be a random sample of size '$n$' from two parameter FD, then the likelihood of function of equation (2) is:

$$L = \prod_{i=1}^{n}\left(\frac{\alpha}{\beta}\right)\left(\frac{x_i}{\beta}\right)^{-\alpha-1} exp^{-\left(\frac{x_i}{\beta}\right)^{-\alpha}} \tag{4}$$

The log-likelihood function is:

$$\ln L = n\ln\left(\frac{\alpha}{\beta}\right) - (\alpha+1)\sum_{i=1}^{n}\ln\left(\frac{x_i}{\beta}\right) - \sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{-\alpha} \tag{5}$$

Partially differentiating equation (5) with respect to α and β and equating it to zero the score equations are

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n}\ln\left(\frac{x_i}{\beta}\right) + \sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{-\alpha}\ln\left(\frac{x_i}{\beta}\right) \tag{6}$$

$$\frac{\partial \ln L}{\partial \beta} = -\frac{n}{\beta} + (\alpha+1)\sum_{i=1}^{n}\frac{1}{\beta}\left(\frac{x_i}{\beta}\right)^{-\alpha} \tag{7}$$

Laplace approximation is employed to obtain the MLEs and the BEs of FD as aforementioned equations cannot be extracted in closed form.

- **BE for FD**

The BE for α and β based on the reference prior, which was developed by [20] i.e.,

$$\prod_R(\alpha,\beta) \propto \frac{1}{\alpha\beta}\ \alpha,\beta > 0\ , \tag{8}$$

Then the joint posterior distribution for α and β is

$$\prod(\alpha,\beta|x) = \frac{\alpha^{n-1}\beta^{n\alpha-1}\prod_{i=1}^{n}x_i^{-(\alpha+1)}exp\left\{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}\right\}}{\int_0^{\infty}\int_0^{\infty}\alpha^{n-1}\beta^{n\alpha-1}\prod_{i=1}^{n}x_i^{-(\alpha+1)}exp\left\{-\sum_{i=1}^{n}\left(\frac{\beta}{x_i}\right)^{\alpha}\right\}d\alpha d\beta}\ . \tag{9}$$

BEs of FD is obtained by using the Laplace approximation.

**4.2 LLD**

The LLD is a continuous probability distribution for a non-negative random variable. Although it is similar to log-normal distribution in shape but contains a heavier tail. The PDF and CDF of LLD are

$$f(x;\alpha,\beta) = \frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{x}{\alpha}\right)^{\beta-1}}{\left[1+\left(\frac{x}{\alpha}\right)^{\beta}\right]^2},\quad x > 0,\ \alpha,\beta > 0 \tag{10}$$

$$F(x) = \frac{1}{1+\left(\frac{x}{\alpha}\right)^{-\beta}} \tag{11}$$

Where α, β are the scale and shape parameters of LLD.

**4.2.1 MLE for LLD**

Let $x_1, x_2, x_3, \dots, x_n$ be a random sample of size '$n$' from LLD, then the likelihood of function of Eq. (10) is:

$$L = \beta^n\alpha^{-n\beta}\prod_{i=1}^{n}(x_i)^{\beta-1}\prod_{i=1}^{n}\left\{1+\left(\frac{x_i}{\alpha}\right)^{\beta}\right\}^{-2} \tag{12}$$

Then the log-likelihood function can be written as:

$$ln\ L = nln\ (\beta) - n\beta\ ln\ (\alpha) + (\beta-1)\sum_{i=1}^{n}ln\ x_i - 2\sum_{i=1}^{n}ln\ x_i + \left\{1+\left(\frac{x_i}{\alpha}\right)^{\beta}\right\} \tag{13}$$

Partially differentiating (13) with respect to α and β and equating it to zero.

$$\frac{\partial \ln L}{\partial \alpha} = -\frac{n\beta}{\alpha} + \frac{2\beta}{\alpha}\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta} ln\ \left\{1+\left(\frac{x_i}{\alpha}\right)^{\beta}\right\}^{-1} = 0 \tag{14}$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} - nln\ \alpha + \sum_{i=1}^{n}ln\ x_i - 2\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta}\ln\left(\frac{x_i}{\alpha}\right) = 0 \tag{15}$$

The above equations cannot be extracted in closed form. So, the Broyden Fletcher Gold Farb Shanno (BFGS), which is available in the R software, have been used to estimate α and β.

**4.2.3 BE for LLD**

BE for LLD is obtained by considering the reference prior, which was developed by Abbas and Tang (2016). Then the joint posterior distribution of α and β is:

$$\prod(\alpha,\beta|x) = \frac{\alpha^{-n\beta-1}\beta^{n-1}\prod_{i=1}^{n} x_i{}^{\beta-1}\prod_{i=1}^{n}\left\{1+\left(\frac{x_i}{\alpha}\right)^{-\beta}\right\}^{-2}}{\int_0^\infty\int_0^\infty \alpha^{-n\beta-1}\beta^{n-1}\prod_{i=1}^{n} x_i{}^{\beta-1}\prod_{i=1}^{n}\left\{1+\left(\frac{x_i}{\alpha}\right)^{-\beta}\right\}^{-2} d\alpha d\beta}. \tag{16}$$

Here we use Metropolis within Gibbs sampling algorithm to obtain the BEs of LLD.

**4.3 WD**

WD is a continuous probability distribution which was proposed by [21]. The PDF and CDF of WD are

$$f(x;\,\alpha,\beta) = \left(\frac{\beta}{\alpha}\right)\left(\frac{\beta}{\alpha}\right)^{\beta-1} exp^{-\left(\frac{x}{\alpha}\right)^{\beta}},\ x>0,\ \alpha,\beta>0, \tag{17}$$

And the corresponding CDF of WD is

$$F(x) = 1-exp^{-\left(\frac{x}{\alpha}\right)^{\beta}}, \tag{18}$$

Where β is the shape and α is the scale parameters of WD.

**4.3.1 MLE of WD**

The likelihood function is

$$L = \alpha^{n}(\beta)^{-n\alpha}\prod_{i=1}^{n} x_i{}^{\alpha-1}\, exp^{-\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha}}, \tag{19}$$

The log likelihood function of equation (19) is

$$ln\, L = nln\,(\alpha) - n\alpha ln\,(\beta) + (\alpha-1)\sum_{i=1}^{n}\left\{\left(\frac{x_i}{\beta}\right)^{\alpha} ln\,\left(\frac{x_i}{\beta}\right)\right\} = 0 \tag{20}$$

Partially differentiating equation (20) with respect to 'α' and 'β' and equating it to zero.

$$\frac{\partial ln\,L}{\partial\alpha} = \frac{n}{\alpha} - nln\,(\beta) + \sum_{i=1}^{n}\left\{\left(\frac{x_i}{\beta}\right)^{\alpha} ln\,\left(\frac{x_i}{\beta}\right)\right\} = 0 \tag{21}$$

$$\frac{\partial ln\,L}{\partial\beta} = -\frac{n\alpha}{\beta} + \left(\frac{\alpha}{\beta}\right)\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha} = 0 \tag{22}$$

The above equations (21) and (22) cannot be written in closed form, so an iterative method will be used to get the ML estimates of α and β.

**4.3.2 BE of WD**

To get the Bayesian estimates of WD, non-informative priors will be used. The joint posterior distribution of $\alpha$ and $\beta$ is

$$\pi\left(\alpha,\ \beta\,\middle|\,x\right) = \frac{\beta^{n-1}\alpha^{-(n\beta+1)}\prod_{i=1}^{n} x_i{}^{(\beta-1)}\ exp\left\{-\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta}\right\}}{\int_0^\infty\int_0^\infty \beta^{n-1}\alpha^{-(n\beta+1)}\prod_{i=1}^{n} x_i{}^{(\beta-1)}\ exp\left\{-\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta}\right\}d\alpha d\beta}. \tag{23}$$

Here, Laplace approximation is used to get the Bayesian estimates for WD.

## 4. Models Selection

Many other tests for model selection can be used. Here the Kolmogrove Smirnov (KS) test is used for the selection of best fitted distributions.

**5.1 KS Test**

Kolmogorov Smirnov test [22] is used to find the best-fit distribution(s) at 5 percent level of significance, which can be calculated as

$$D = max(D^{+}, D^{-}),$$

$$D^{+} = max\left(\frac{i}{n} - F(x_i)\right),$$

$$D^{-} = max\left(F(x_i) - \frac{i-1}{n}\right),$$

## 5. Results And Discussions

Table 1 displays the descriptive statistics for annual maximum data of WS at height 10m and 50m respectively. On average the highest mean value of WS is at 10m and 50m are 10.4440 and 12.3940 respective. The coefficient of skewness (-0.1917 for 10m and-0.09675 for 50m) indicate that the data is negatively skewed for the height 10m and 50m. The kurtoses (2.6499 for 10m and 2.3962 for 50m) depict that the distribution of data is platykurtic.

**Table 1:** Descriptive statistics for annual maximum data of WS

| Height | $n$ | Mean | Median | S.D | Min. | Max. | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|---|
| WS10m | 29 | 10.4440 | 10.46 | 1.0581 | 7.9000 | 12.3000 | -0.1917 | 2.6499 |
| WS50m | 29 | 12.3940 | 12.49 | 1.2141 | 9.7500 | 14.6500 | -0.09675 | 2.3962 |

***Note: n= sample size; Min= Minimum; Max= Maximum; S.D = Standard deviation.***

### 5.2 Testing of Basic Assumption of Wind Speed

Before fitting the distributions to the data set, we check the randomness of the observed data set using the run test. P-values of WS at 10m and 50m, which are greater than 0.05, we accept the hypotheses of randomness.

**Table 2:** Run tests 10m and 50m

| Variables | $n$ | Runs | P-value |
|---|---|---|---|
| WS 10m | 28 | 14 | 0.7001 |
| WS 50m | 28 | 14 | 0.7001 |

### 5.3 Parameter Estimation of WS

FD, WD and LLD are fitted to WS data and the parameters of these distributions were estimated by using MLE and BE methods along with KS test and results are presented in Tables 3 and 4.

**Table 3:** Estimated parameters of FD, WD and LLD for WS at Height 10m

| **FD** | | | | |
|---|---|---|---|---|
| **Methods** | $\hat{\alpha}$ | $\hat{\beta}$ | **KS** | **P-values** |
| **MLE** | **9.9385** | **11.7337** | **0.1160** | **0.7883** |
| **BE** | **9.7674** | **11.7390** | **0.1145** | **0.8004** |
| **WD** | | | | |
| **MLE** | **11.6177** | **12.9344** | **0.1204** | **0.7505** |
| **BE** | **11.3718** | **12.9201** | **0.1138** | **0.806** |
| **LLD** | | | | |
| **MLE** | **17.62870** | **12.3706** | **0.0711** | **0.9962** |
| **BE** | **17.2090** | **12.3656** | **0.0674** | **0.9982** |

**Table 4:** Estimated parameters of FD, WD and LLD for WS at Height 50m

| **FD** | | | | |
|---|---|---|---|---|
| **Methods** | $\hat{\alpha}$ | $\hat{\beta}$ | **KS** | **P-values** |
| **MLE** | **8.9990** | **9.85635** | **0.1274** | **0.7343** |
| **BE** | **8.8571** | **9.8611** | **0.1317** | **0.6958** |
| **WD** | | | | |
| **MLE** | **11.3741** | **10.9115** | **0.0981** | **0.9427** |
| **BE** | **11.1316** | **10.8984** | **0.0892** | **0.9751** |
| **LLD** | | | | |
| **MLE** | **17.1550** | **10.4241** | **0.0771** | **0.9953** |
| **BE** | **16.7539** | **10.4200** | **0.0721** | **0.9982** |

Tables 3 and 4 shows the results of FD, WD and LLD for WS at heights of 10m and 50m respectively. The P values show that all three distributions are good fitted to the WS data.

### 5.4 PDF Graphs for WS

The PDF graphs are employed for comparison purposes of estimation methods. PDF graphs for annual maximum WS at height 10m and 50m are shown in Figures 1-2. Graphs demonstrate that MLE and BE methods show almost same behavior for FD, WD and LLD.

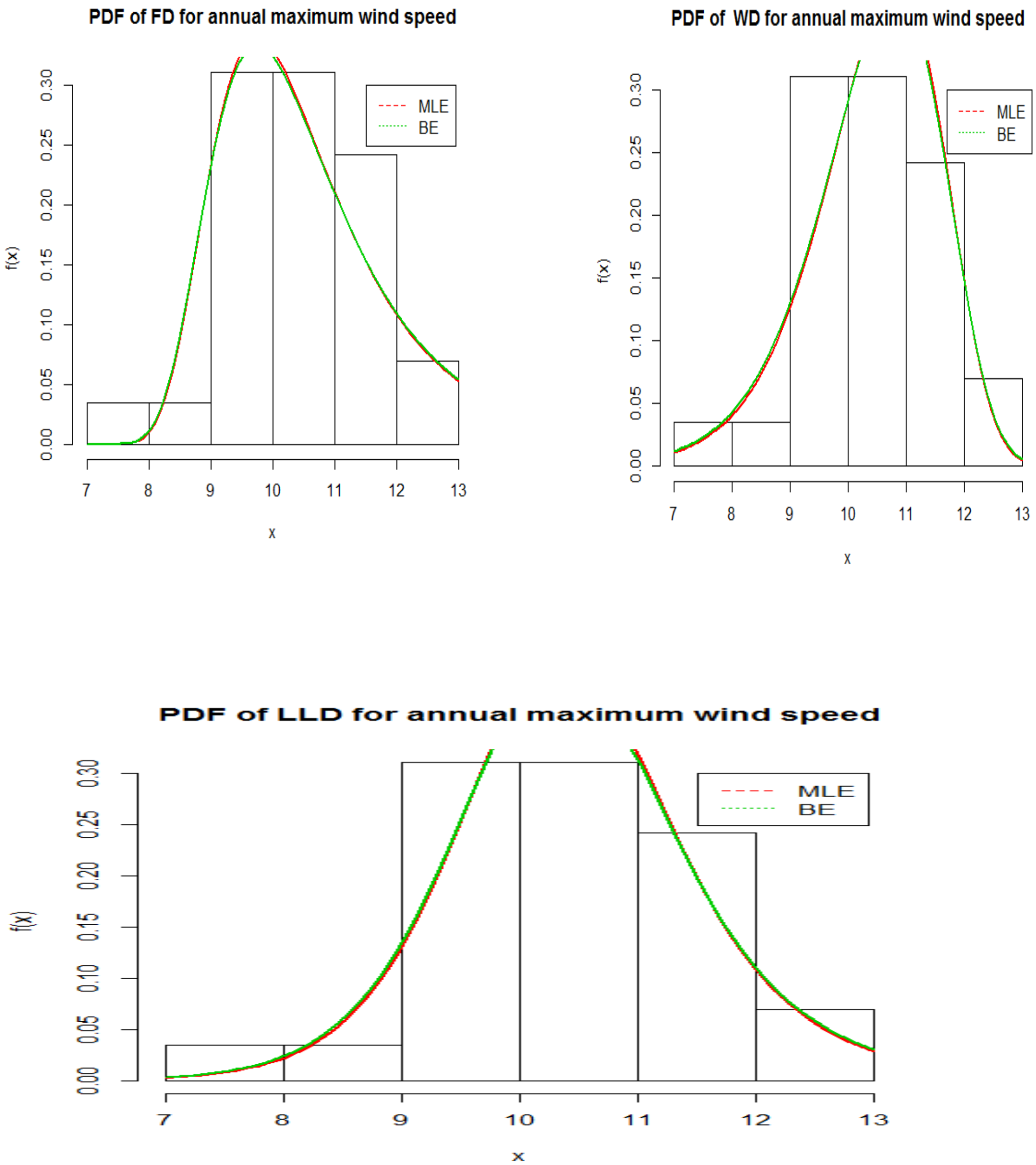


***Fig. 1.*** *PDF graphs of WS data at Height 10m*

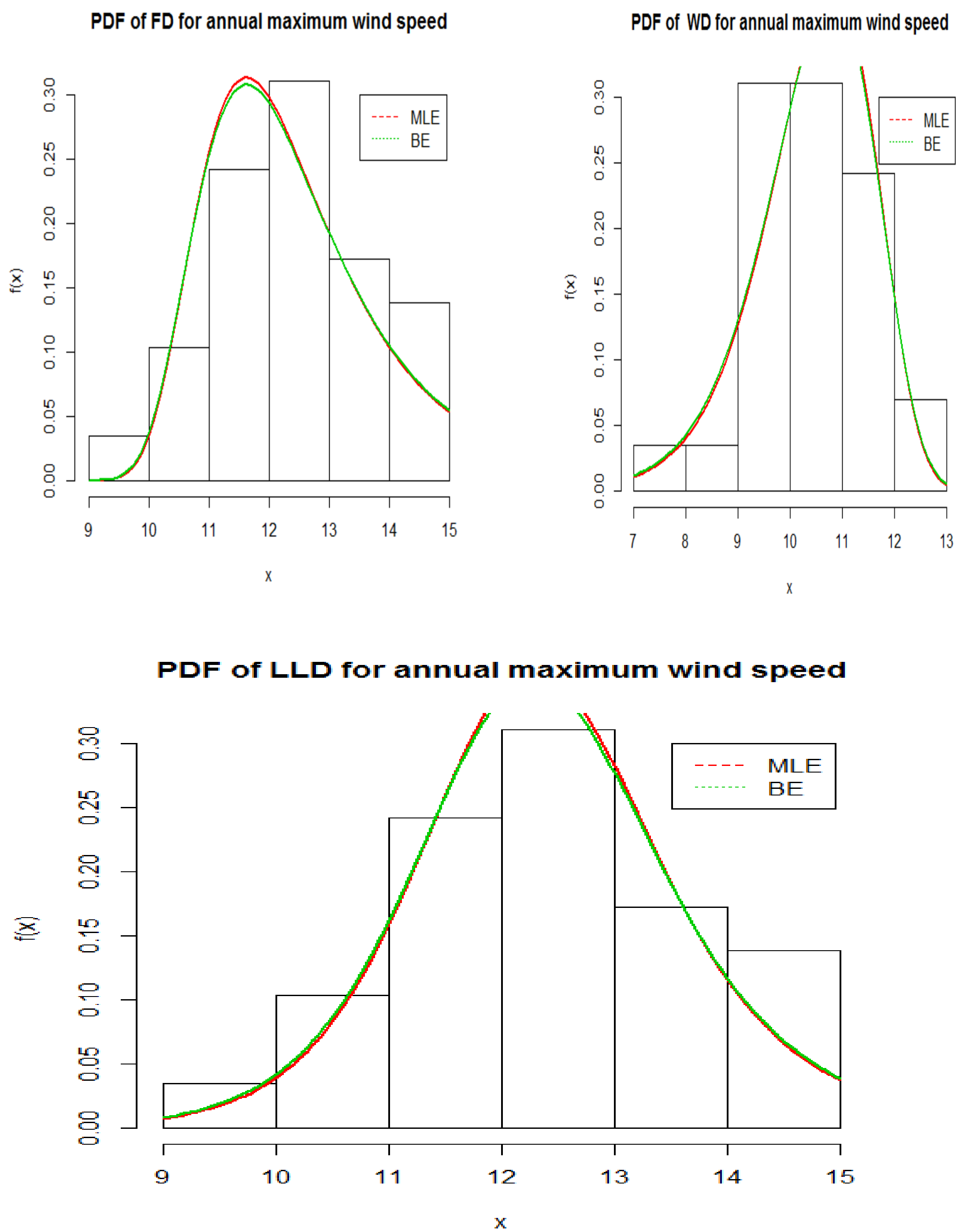


***Fig. 2.*** *PDF graphs of WS data at Height 50m*

## 6 Analysis of Wind Direction Data

The analysis of annual maximum Wind Direction data also conducted. Table 5 showed the descriptive statistics at height 10m and 50m. On average the highest mean value of Wind Direction is 358.47 at 10m. Coefficients of skewness indicate that the distribution of data (-1.2702 for 10m and -2.0566 for 50m) is skewed at the direction of 10m and 50m. The coefficients of kurtosis depict that the distribution of data is leptokurtic.

**Table 5:** Basic statistics for annual maximum data of Wind Direction

| Height | $n$ | Mean | Median | S.D | Min. | Max. | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|---|
| Wind Direction 10m | 29 | 358.47 | 359.1 | 1.5219 | 354.0700 | 359.9800 | -1.2702 | 3.8128 |
| Wind Direction 50m | 29 | 357.81 | 358.1 | 2.2762 | 349.300 | 359.9400 | -2.0566 | 7.8116 |

### 6.1 Randomness of Wind Direction Data

To verify the randomness of the pragmatic data series we use run test. Table 6 demonstrates that Wind Direction follows the assumption of randomness.

**Table 6:** Runs and P-value for Wind Direction

| Height | n | Runs | P-value |
|---|---|---|---|
| Wind Direction 10m | 28 | 14 | 0.7001 |
| Wind Direction 50m | 28 | 14 | 0.4410 |

### 6.2 Parameter Estimation of Wind Direction Data

The following tables show the estimated parameters of the selected distributions for maximum annual Wind Direction at height 10m and 50 m.

**Table 7:** Estimated parameters of FD, WD and LLD for Wind Direction at height 10m

| FD | | | | |
|---|---|---|---|---|
| **Methods** | $\hat{\alpha}$ | $\hat{\beta}$ | **KS** | **P-values** |
| **MLE** | 196.5856 | 357.6362 | 0.2419 | 0.0672 |
| **BE** | 193.4297 | 357.6510 | 0.2479 | 0.0566 |
| **WD** | | | | |
| **MLE** | 367.4877 | 359.1025 | 0.1603 | 0.4452 |
| **BE** | 359.7094 | 359.0934 | 0.1642 | 0.4151 |
| **LLD** | | | | |
| **MLE** | 441.3711 | 358.7025 | 0.1723 | 0.3551 |
| **BE** | 430.5240 | 358.6962 | 0.1768 | 0.3249 |

**Table 8:** Estimated parameters of FD, WD and LLD for Wind Direction at height 50m

| FD | | | | |
|---|---|---|---|---|
| **Methods** | $\hat{\alpha}$ | $\hat{\beta}$ | **KS** | **P-values** |
| **MLE** | 110.2825 | 356.4961 | 0.2928 | 0.0139 |
| **BE** | 108.7901 | 356.5195 | 0.2981 | 0.0116 |
| **WD** | | | | |
| **MLE** | 256.7994 | 358.6578 | 0.1838 | 0.2814 |
| **BE** | 265.7725 | 358.6821 | 0.1739 | 0.3445 |
| **LLD** | | | | |
| **MLE** | 328.1695 | 358.1878 | 0.1717 | 0.3596 |
| **BE** | 319.5977 | 358.1799 | 0.1730 | 0.3504 |

The estimates of parameters of FD, WD and LLD on the annual maximum Wind Direction at height 10m and 50m are shown in Tables 7-8. The results indicate that all three distributions are best fitted with both methods at the P-values of KS test at 10m and 50m.

### 6.3 PDF Graphs of Wind Direction

PDF graphs for annual maximum Wind Direction at heights 10 and 50m are shown in Figures 3 and 4. Graphs demonstrate that MLE and BE methods show almost the same behavior for FD, WD and LLD.

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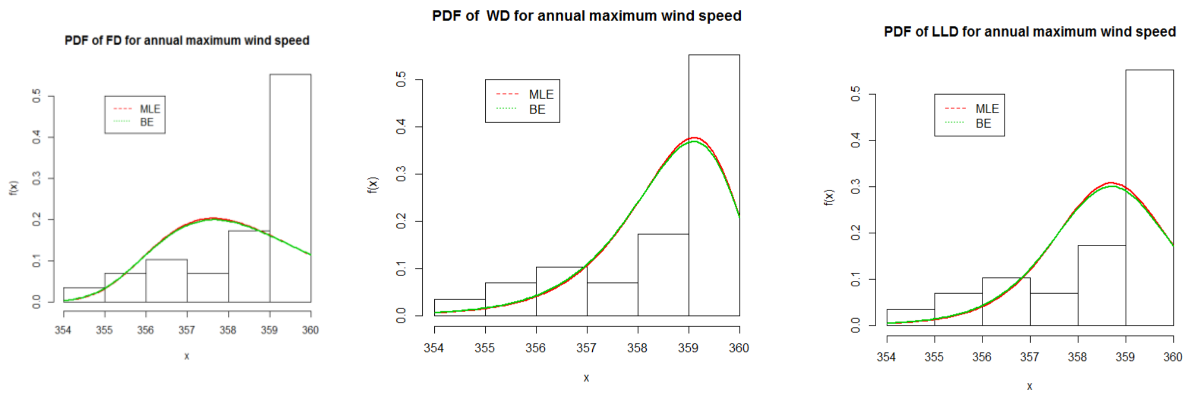



***Fig. 3.*** *PDF Graphs for Wind Direction at Height 10m*

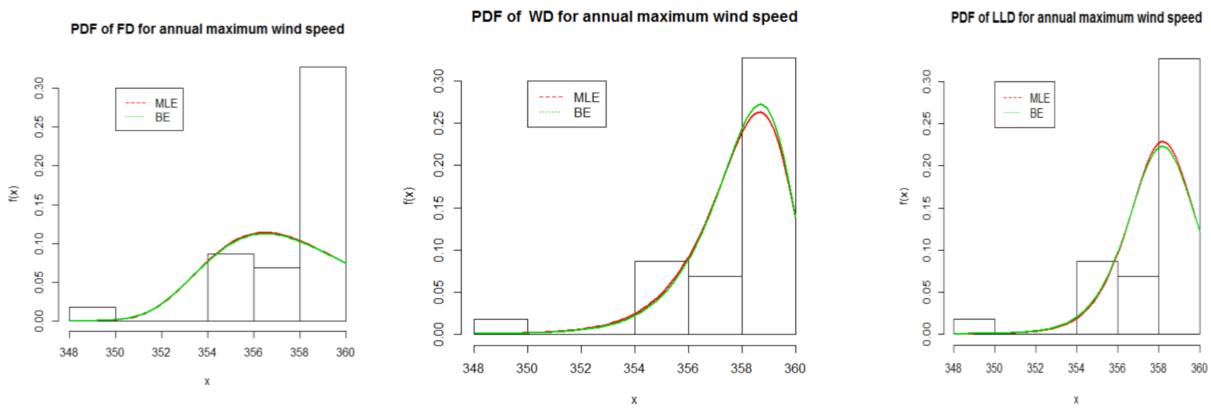



***Fig. 4.*** *PDF Graphs for Wind Direction at Height 50m*

### 1. PDF Graphs Year Wise Comparison

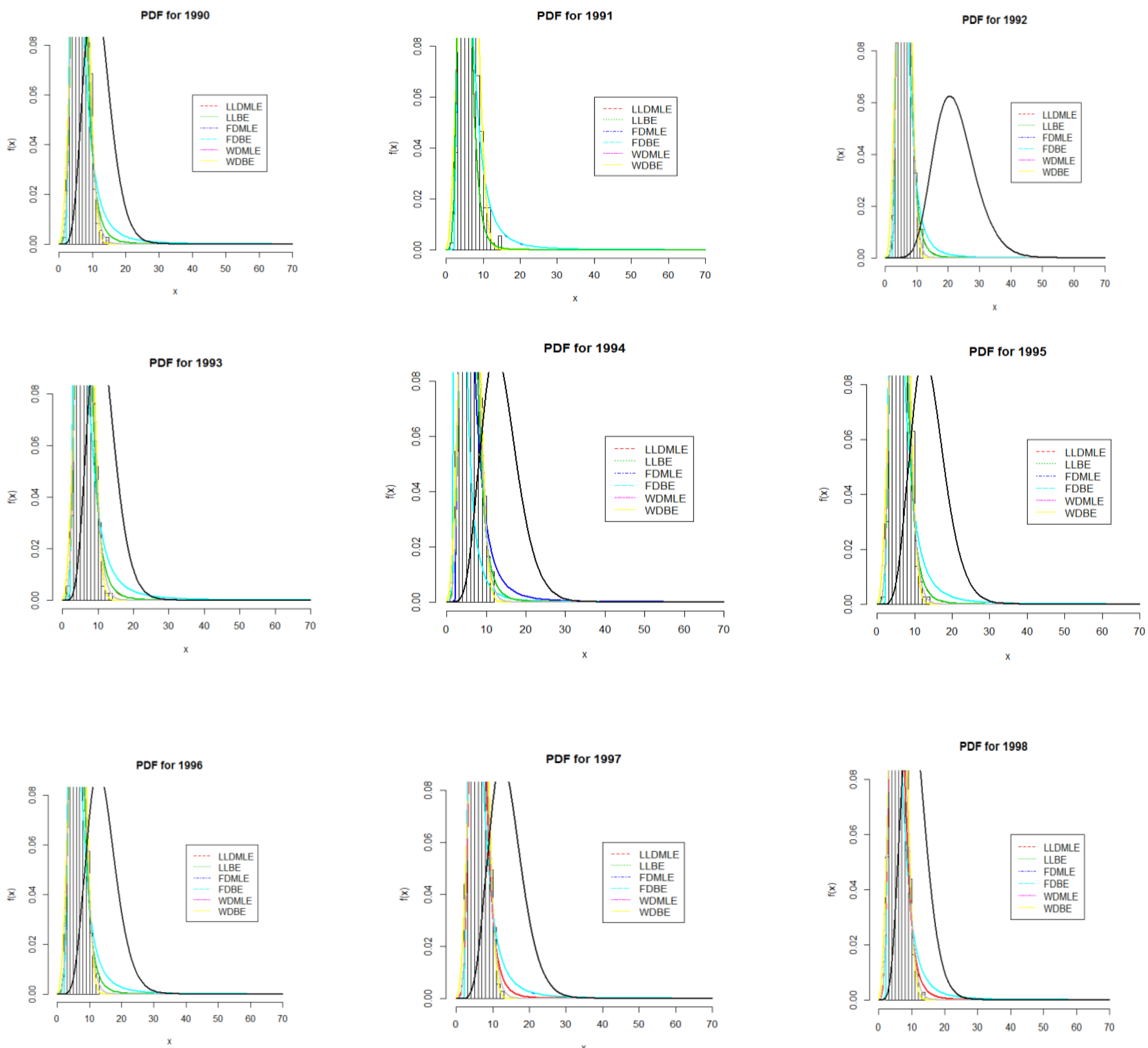
The year-wise PDF graphs are constructed to check the behavior of FD, WD, and LLD and are shown in Figure 5. PDF Graphs demonstrate that MLE and BE methods show almost the same behavior for FD, WD, and LLD for WS year-wise data. From Figure 5, it is clear that the above selected distributions can be used to estimate that kind of data.

### 8. Comparison

The performance of different distributions for WS and Wind Direction data has been assessed. For this purpose, a comparison of FD, WD and LLD distributions with estimation methods such as MLE and BE were Evaluated. In Table 9 the word "Yes" indicates the candidate distributions using both estimation methods are a good fit for observed data whereas the word "No" indicates that the candidate distributions using the selected methods are not appropriate for the observed data.

**Table 9:** Evaluations of Distributions

| Heights | Methods | FD | WD | LLD |
|---|---|---|---|---|
| WS at 10m | MLE | Yes | Yes | Yes |
| | BE | Yes | Yes | Yes |
| WS at 50m | MLE | Yes | Yes | Yes |
| | BE | Yes | Yes | Yes |
| Wind Direction at10m | MLE | Yes | Yes | Yes |




| | | | | |
|---|---|---|---|---|
| | BE | Yes | Yes | Yes |
| Wind Direction at50m | MLE | No | Yes | Yes |
| | BE | No | Yes | Yes |

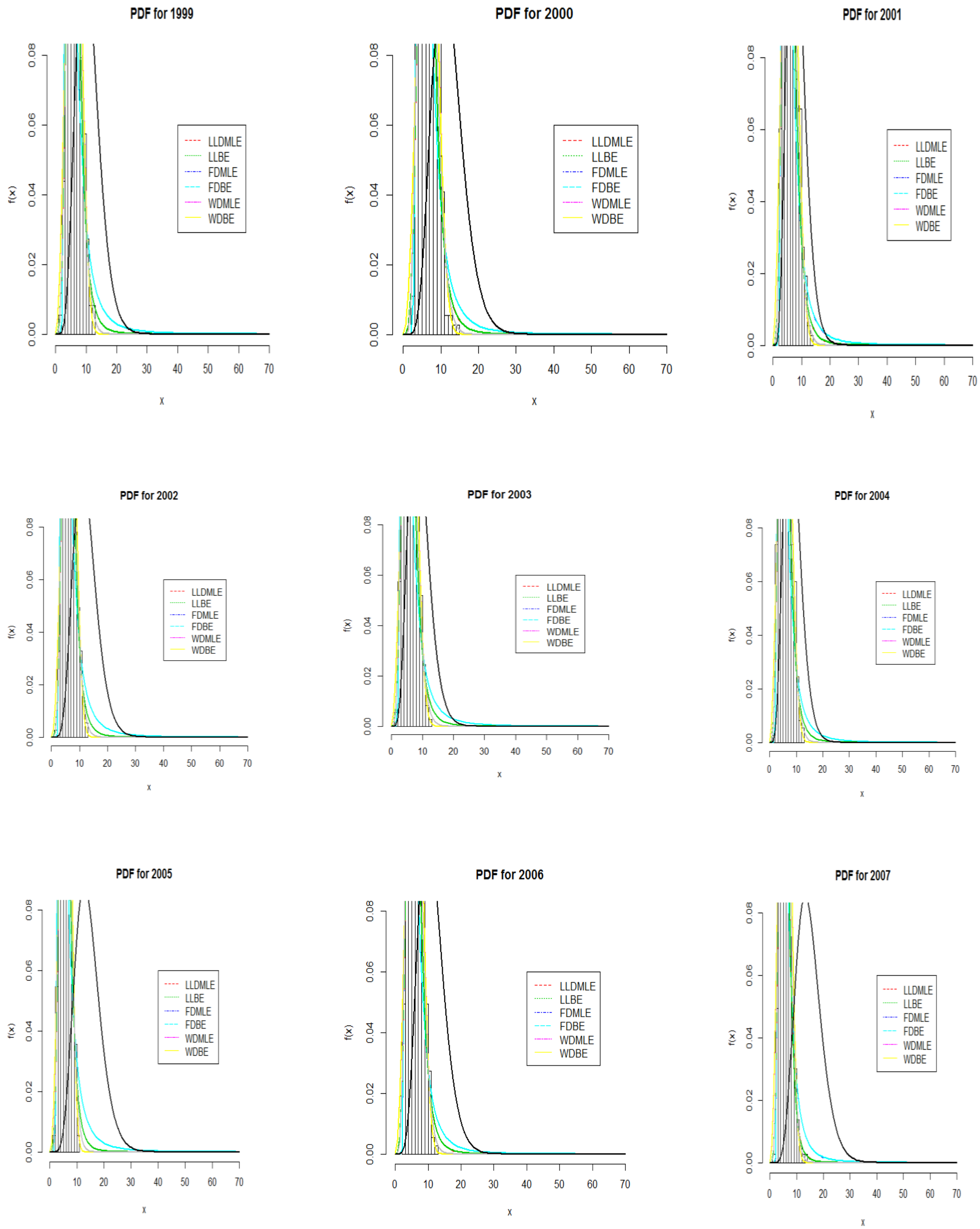
From Table 9 it is clear that FD distribution for Wind direction is not appropriate for 50m height. It is also clear that with both methods, WD and LLD are best-fitted distributions for WS and Wind Direction.

## 9. Conclusions

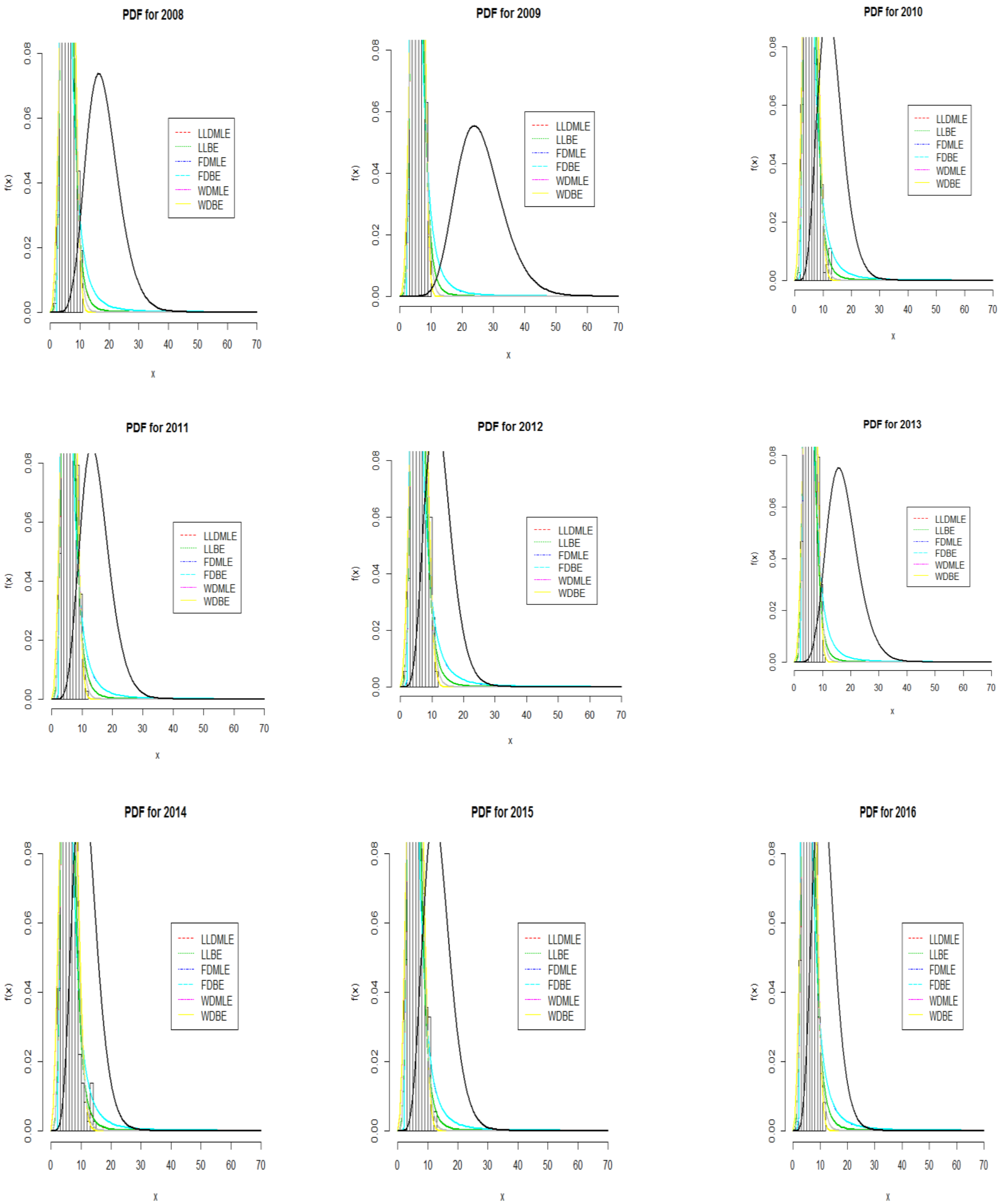
In the current study, WS and Wind Direction data at heights 10m and 50m have been used. Three distributions such as FD, WD, and LLD are applied.

I. Firstly, the basic statistics for annual maximum WS data at heights of 10m and 50m are computed. On average the maximum mean value of WS is 50m. The results of skewness show that the data distribution is negatively skewed. Kurtosis depicts that the data distribution is platekurtic. The Run test is used to check the randomness of data.

II. The primary goal of WS analysis at heights of 10m and 50m is to find the most suitable distribution(s) and select the appropriate method for estimation. MLE and BE estimating methods are employed and the P-values for the annual maximum WS at heights of 10m and 50m show that all three distributions are well suited to the WS data. Furthermore, the PDF Graphs reveal that the MLE and BE techniques exhibit nearly identical behavior for FD, WD and LLD.




III. The analysis of annual maximum Wind Direction data is also conducted for heights 10m and 50m. On average the highest mean value of Wind Direction is 358.47 at 10m. Skewness coefficients imply that




the distribution is skewed towards the direction of 10m and 50m. Kurtosis depicts that the distribution

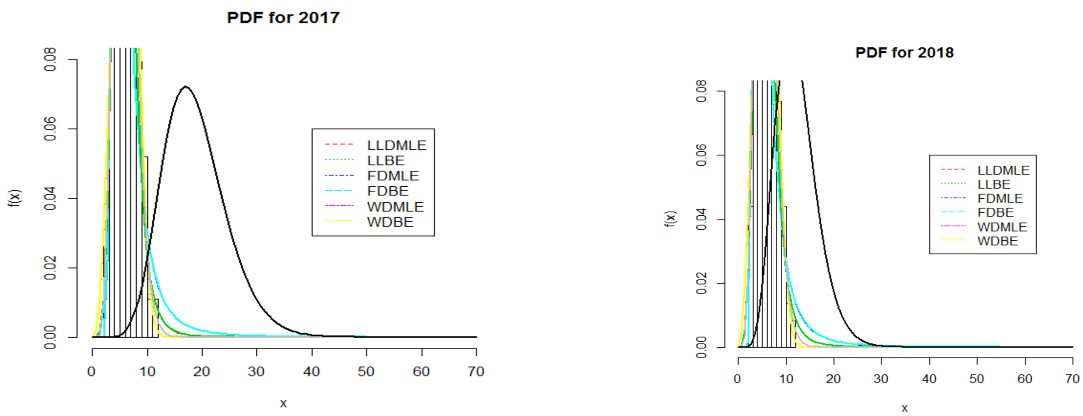



***Fig. 6.*** *PDF Graphs for year Wise Data of WS at Height 50m*

of data is leptokurtic.

IV. The FD, WD and LLD are (with MLE and BE methods) fitted to Wind Direction at height 10m and 50m respectively. The results indicate that only WD and LLD are best fitted with both methods at the P-value of KS test at 10m and 50m. PDF graphs for annual maximum Wind Direction at height 10 and 50m are shown that MLE and BE methods show almost same behavior. It is clear that with both methods (MLE and BE) WD and LLD are best fitted distribution for WS and Wind Direction.

To assess the behavior of MLE and BE methods for WS, the PDF graphs were used from 1990 to 2018. The PDF comparison is only for 50m height because only at 50m graphs show the best similar behavior with both methods. PDF as Graphs demonstrates that MLE and BE methods show almost the same behavior for FD, WD, and LLD for WS year-wise data. Selected distributions can be used to estimate that kind of data. So, it can be suggested that for this kind of data WD and LLD can be used. Finally, it is concluded that WD and LLD are more appropriate for WS and Wind Direction data as compared to FD.

### 9.1 Practical Application

This study provides useful guidelines for the concerned officials of the study area and meteorological studies, this study will be helpful for engineers and scientists to study the WS and Wind Direction for different heights. The procedure can be adopted in general to bring diversity and flexibility in the choice of probability distributions for modeling extremes of events like storms, rainfall, etc.

### 9.2 Recommendations

For the statistical analysis of WS and Wind Direction at heights of 10 and 50 meters using MLE and BE methods, two-parametric distributions were used in the current study. This research could be extended by

- Using further two and three-parameter distributions such as Log Gamma, Beta, Gamma 3 parameter distribution, etc.
- Applying other methods of estimation like Least squares, Moments, and Probability Weighted Moments etc.
- Considering other locations across the country.
- Conducting time series analysis.

### 9.3 Limitations of this study

This study was only for three distributions. A number of distributions that's can be used to study the WS and Wind Direction to extend the study furthermore, here only two methods were used some other methods can also be used like Moments Estimation and Probability Weighted Moments method.

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