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Analysis of Analytical Techniques for Deciphering Fluid Flows under Certain Regime

Aamra Urooj[1], Ehsan Ul-HAq[2], Asad Iqbal[3], Jamshad Ahmad[4]

[1] Govt Associate College for Women, Pindi Gheb, Punjab Higher Education Department, Pakistan.

[2] The Chinese University of Hong Kong , Hong Kong.

[3] Department of Mathematics, University of Poonch Rawalakot, AJ&K.

[4] Department of Physical & Numerical Sciences, University of Rasul, Pakistan.

Corresponding author: aamraurooj321@gmail.com

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Abstract

Complex systems of ordinary and partial differential equations govern the great majority of phenomena in mathematical physics and engineering. By using sophisticated analytical frameworks to solve these governing equations, this study offers a thorough investigation into the dynamics of fluid flow and heat transfer. In particular, the study investigates the coupled effects of mass and heat transfer over an inclined stretching sheet in the magneto-hydrodynamic (MHD) flow of a non-Newtonian Burgers' fluid through a porous medium. Two reliable analytical techniques, the Variational Iteration Method (VIM) and the Variational of Parameter Method (VPM), are used to preserve high accuracy while guaranteeing computational efficiency. These techniques offer sophisticated, closed-form approximations that avoid the high computational costs of conventional numerical grid-based simulations. The impact of different physical parameters on the temperature and velocity distributions is examined in detail, and graphical representations are offered to confirm the dependability and consistency of the suggested analytical models. The results provide a theoretical foundation for understanding the sensitivity of flow parameters, which is essential for the future design and optimization of transport mechanisms in MHD-based industrial systems.

Keywords: *Analytical Techniques, Variational Iteration Method, Variational Perturbation Method, Burger's Fluid*

1. Introduction

In the last few decades, there is a rapid development in non-Newtonian fluids rather than the Newtonian fluids because of its several applications in industry [19]. Polymer, ketchup, toothpaste, starch suspension, blood and gelatin etc. are few examples of non-Newtonian fluids. These fluids are further categorized into differential, integral and rate type equations. Most phenomena in fluid mechanics are described by differential equations. To analyze fluid behavior, these nonlinear equations are solved using various analytical techniques, including the Homotopy Analysis method [14], Variational iteration method [8-11], Differential transform method [13], Variation of parameters method [21-22] and their modifications. We used the Variational iteration method and Variation of parameters method for different fluid problems and presented the results graphically.

As described earlier, non-Newtonian fluid are more important. In this context, number of research work is present to study the flow induced by a stretching sheet. Such type of flows has many applications like hot rolling, paper production etc. Few attempts with the constant thermal conductivity are Exact solution to generalized Burger's fluid is studied by [28]. An important rate type non-Newtonian fluid in three dimensions is examined by Hayat et al. [25-26]. An important concept of variable thermal conductivity was taken into account first time by Chiam [23-24]. Recently Oyem [17] discussed the influence of variable thermal conductivity for incompressible flow of a viscous fluid. We discussed the impact of heat and mass transfer on the Burger's fluid over stretching sheet inclined with x-axis and presented the results for various parameters involved in governing differential equations on velocity and temperature field graphically.

An important phenomena arising in physical sciences is diffusion. It is also popular in well-testing world. Problems of multi-dimensional, multi-phase have never been solved for well-testing. A partial differential equation describing the flow through porous media generally developed by combination of equation of continuity and the Darcy's law [7] and known as diffusivity equation. Therefore, when the typical non-dimensionless groups of well-testing problems are imported into diffusivity equation, a set of non-dimension, non-linear PDEs are obtained. We applied the two analytical methods named Variational iteration and Variation of parameters method to solve a diffusivity equation in simplest form. Both the techniques are analytical and proved to be accurate and reliable.

'Magento-Hydro Dynamics' that is the study of magnetic properties of fluids that are electrically conductors. An important application of MHD is the interaction of electrically conductor's fluids and electromagnetic field is.The flow of Magneto-Hydrodynamics fluid between parallel plates is knowns as squeezing flow. An MHD fluid work as lubricant. Maki et al. [5] experimented the Magento-Hydro dynamics lubrication in an superficially pressurized thrust bearing and presented the results theoretically. In lubrication, magnetic effects are studied in [4] and [27]. Alabdulhadi et al. [28] carried out numerical analysis of MHD mixed convection flow of a viscous fluid over an inclined stretching sheet. Latest Studies also explored the dynamics of MHD flow of Burger's fluid prompted by a stretching container, underscoring the impacts of internal heat generation and absorption [29]. Mustafa et al. [30] implemented modified VIM with a genetic algorithm solving nonlinear partial differential equations. Mamadu et al. [31] employed VIM for deciphering the nonlinear Burgers Equation's dynamics. In present study, we considered a squeezing flow of a magneto-hydrodynamic fluid in two dimensions between moving parallel plates.examine the behavior of this fluid's problem. Behaviors of dimensionless parameters involved in fluid model are discussed graphically in detail.

The salient features of this research study are as follows:

- Concentrates on Burgers' fluid model, which is more complicated than conventional Newtonian models because it takes into account both elasticity and viscosity.
- Examines how magnetic fields affect electrically conducting fluid flow
- Examines the fluid's interaction with a porous structure, which is important for geothermal and filtration research.
- Adds gravitational and directional complexity to the boundary layer analysis by examining flow over an inclined stretching sheet.
- Discusses the relationship between heat and mass transfer at the same time (thermophysical coupling).
- Uses both the Variational Perturbation Method (VPM) and the Variational Iteration Method (VIM).
- These methods provide high-precision, **closed-form approximations** without the heavy computational "overhead" required by traditional grid-based numerical simulations.

The rest of the article is arranged in the following manner: Section 2 includes analysis of method, section 3 portrays the problem development, section implements analytivcal technique to the problems discussedin section 3, section5 discusses the findings, section 6 conslude the study, and section7 cites the refernces for this study.

2. Materials and Methods

It is well known that the greater number of issuesin science and engineering are linear and non-linear. Particularly the problems of fluid mechanics are highly non-linear.These problems tend to be more difficult to solve either numerically or analytically. Numerous method, therefore, were proposed to solve these problems. Among these VIM and VPM is the most used and simple analytical techniques. We have used these analytical techniques to obtain the analytical solution to fluids' problems.

2.1 Variational Iteration Method

The variational iteration method is the most comprehensive, simple and user friendly technique to solve the differential equations. For the first time introduced by JI-Huan He in 1999 [8-9]. It has been extensively used by many authors to solve problems with high non-linearity. He used this technique for approximate solutions for non-linear differential equations.

Consider the general differential equation as:

$$Lv + Nv = g(x), \tag{2.1}$$

In above equation v is unknown function which is to be determined, L is linear operator and N are linear and operator, and $g(x)$ is the inhomogeneous term. The correction functional for above equation [8-11] is given by

$$v_{n+1} = v_n(x) + \int_0^x \lambda(t)\left[Lv_n(t) + Nv_n(t) - g(t)\right]dt, \tag{2.2}$$

Where λ is Lagrange's multiplier and it can be a constant or a functions.

In this method, first we determine the value of Lagrange multiplier which can be determined optimally via integration by parts and by using restricted variation. By using the value of Lagrange multiplier [2] determine the successive approximations $v_{n+1}(x)$ of the solution $v(x)$. The zeroth ordered approximation $v_0(x)$ can be any selective function. Finally the solution is given by

$$v(x) = \lim_{n\to\infty} v_n(x). \tag{2.3}$$

2.2. Variation of Parameter Method

The Variation of parameter method was very firstly introduced in (1707-1783) by a Swiss-born mathematicians named Leonard Euler and then an Italian- French mathematician Joseph-Louis Lagrange completed in (1736-1813). Lagrange gives the final form of the VPM during 1808-1810.

VPM is also known as variation of constants in mathematics. This method is generally used for solving inhomogeneous linear ordinary differential equations ODEs.

Consider a non-homogeneous differential equation of finite order,

$$Lv + Rv + Nv = g(x), \tag{2.4}$$

Where Lv indicates the highest order linear operator, Rv indicates the linear operator of order lesser than Lv, Nv the nonlinear operator and $g(x)$ represents the source term. Rearranging Equation (2.4) as

$$Lv = g(x) - Rv - Nv, \tag{2.5}$$

By applying VPM, the solution of Equation (2.6) is evaluated as $v(x) = \sum_{i=1}^{k-1} \frac{C_i x^i}{i!} + \int_0^x (\lambda(\xi)(gv - Rv - Nv))d\xi$

(2.7)

Where k and c_i's are the order of equation and obtained by using initial and boundary conditions.

Firstly, we find the multiplier λ and the finally the iterative algorithm obtained is

$$v_{n+1}(x) = v_0(x) + \int_0^x (\lambda(\xi)(gv - Rv - Nv))d\xi \qquad (2.8)$$

Where,

$n = 0,1,2,3,...$

3. Magneto-Hydrodynamics Fluid in Two Dimensions: Squeezing Flow

An important type of flow is an incompressible viscous and squeezing flow between two parallel plates. Various machines and hydro-dynamic tools are working on these phenomena. The squeezing flow has several applications in food industries, chemical engineering, polymer processing etc. In nineteen century, the modeling and analysis of squeezing flow become a great interest for engineers due to its vast applications in physical science and biophysics.

3.1 Mathematical Formulation

In this study, particularly, we considered the unsteady magneto-hydrodynamics squeezing 2D viscous fluid flow between two parallel plates. This problem was originally considered in Siddiqui, Irum and Ansari [1]. Here, central axis is x-axis and Y-axis is perpendicular to it.. The distance between the parallel plates is $2s(t)$. A uniform magnetics field B_0 acting along y-axis. Induced magnetics field is supposed to be negligible. $B_0 = H_0\mu_0$ Where H_0 is constant strength of magnetics field and μ_0 magnetic permeability. The plates are supposed to move symmetrically about central axis.

The equations governing the flow and heat transfer of Magneto-hydrodynamic fluid in two dimensions are given as:

$$\frac{\partial v}{\partial x} + \frac{\partial w}{\partial y} = 0, \qquad (3.1)$$

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) + \frac{\partial p}{\partial x} - V\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \sigma B_0^2 v = 0, \qquad (3.2)$$

$$\rho\left(\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial y}\right) + \frac{\partial p}{\partial y} - V\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0, \qquad (3.3)$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} + w\frac{\partial T}{\partial y} - \alpha\frac{\partial^2 T}{\partial y^2} - \frac{\mu}{\rho c_p}\left(\frac{\partial v}{\partial y}\right)^2 = 0, \qquad (3.4)$$

Where, v and w are the components of velocity along x-axis and y-axis respectively, $V = \left(\frac{\mu}{\rho}\right)$ denotes viscosity, σ the electrical conductivity of the fluid, c_p denotes the specific heat and $\alpha = \frac{k}{\rho c_p}$.

The generalized pressure h and vorticity function ϖ is defined by [20];

$$h = \frac{1}{2}\rho\left(v^2 + w^2\right) + p \qquad (3.5)$$

$$\varpi = \frac{\partial w}{\partial x} - \frac{\partial v}{\partial y} \qquad (3.6)$$

Using equations (3.5) -(3.6) in Equations(3.2) -(3.3) respectively, yields;

$$\frac{\partial h}{\partial x}+\rho\left(\frac{\partial v}{\partial t}-w\varpi\right)=V\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\sigma B_0^2 v \tag{3.7}$$

$$\frac{\partial h}{\partial y}+\rho\left(\frac{\partial w}{\partial t}+v\varpi\right)=V\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \tag{3.8}$$

By applying $\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}=0,$ and vorticity function ϖ , yields;

$$\frac{\partial \varpi}{\partial y}=\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2} \tag{3.9}$$

And

$$\frac{\partial \varpi}{\partial x}=\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2} \tag{3.10}$$

$$\frac{\partial h}{\partial x}+\rho\left(\frac{\partial v}{\partial t}-w\varpi\right)=-V\frac{\partial \varpi}{\partial x}-\sigma B_0^2 v_y \tag{3.11}$$

And

$$\frac{\partial h}{\partial y}+\rho\left(\frac{\partial w}{\partial t}+v\varpi\right)=V\frac{\partial \varpi}{\partial x} \tag{3.12}$$

$$\rho\frac{\partial w}{\partial t}+\rho\left(v\frac{\partial \varpi}{\partial x}+w\frac{\partial \varpi}{\partial x}\right)=V\,\nabla^2\varpi+\sigma B_0^2 v_y \tag{3.13}$$

Now, we define the dimensionless velocity and temperature components [8,18] as follows:

$$v=\frac{\left(C-x\right)w_{\varpi}(t)g'(\xi)}{s(t)},\, w=w_{\varpi}(t)g(\xi),\varpi=-\frac{\left(C-x\right)w_{\varpi}(t)g''(\xi)}{s(t)^2},$$
$$\theta(\xi)=\frac{T-T_{\infty}}{T_{\varpi}-T_{\infty}} \tag{3.14}$$

In equation (4.14), C is a constant, $w_{\varpi}(t)$ is the velocity of the plates, g' is the derivative of velocity with respect to ξ which are introduced in by Alsaedi et al. [10], T_{ϖ} is the temperature of the sheet and T_{∞} is the temperature of the fluid far away from the sheet.

On substitution,

$$\frac{\partial \varpi}{\partial x}=\frac{w_{\varpi}(t)g''(\xi)}{s(t)^2} \tag{3.15}$$

$$\frac{\partial^2 \varpi}{\partial x^2}=0 \tag{3.16}$$

$$\frac{\partial \varpi}{\partial y}=-\frac{\left(C-x\right)w_{\varpi}(t)g''(\xi)}{s(t)^3} \tag{3.17}$$

$$\frac{\partial^2 \varpi}{\partial y^2}=-\frac{\left(C-x\right)(w_{\varpi}(t))^2 g^{(iv)}(\xi)}{s(t)^4} \tag{3.18}$$

$$\frac{\partial \varpi}{\partial t}=\frac{\left(C-x\right)\xi(w_{\varpi}(t))^2 g'''(\xi)}{s(t)^3}+\frac{2\left(C-x\right)(w_{\varpi}(t))^2 g''(\xi)}{s(t)^3}4-\frac{\left(C-x\right)g''(\xi)}{s(t)^2}\frac{dw_{\varpi}}{dt} \tag{3.19}$$

$$\frac{\partial v}{\partial y}=\frac{\left(C-x\right)w_{\varpi}(t)g''(\xi)}{s(t)^2} \tag{3.20}$$

$$\frac{\partial T}{\partial y}=\theta'(\xi)\frac{(T_{\varpi}-T_{\infty})}{s(t)},\quad \frac{\partial^2 T}{\partial y^2}=\theta''(\xi)\frac{(T_{\varpi}-T_{\infty})}{s(t)^2},\tag{3.21}$$

Putting all above values in Equation and Equation and define $R=\frac{\rho s(t)w_{\varpi}(t)}{V}$, $Q=\frac{s(t)}{\rho w_{\varpi}(t)^2}\frac{dw_{\varpi}(t)}{dt}=-\frac{1}{\rho}$, $M=\frac{\sigma B_0^2 s(t)^2}{V}$, $\Pr=\frac{V}{\alpha}$ and $Ec=\frac{v_{\varpi}^2}{c_p(T_{\varpi}-T_{\infty})}$,

$$R\left[gg'''-g'g''-\left(2+\frac{1}{\rho}\right)g''-\xi g'''\right]=g^{(iv)}-M^2 g''\tag{3.22}$$

$$\theta''=\left(-\xi+g\right)\theta'\frac{\Pr R}{\rho}-\Pr Ec g''^2\tag{3.23}$$

Boundary conditions are

$$g(0)=0,\quad g(1)=1,\quad g'(1)=0,\quad g''(0)=0,\quad \theta(0)=1,\quad \theta(1)=0\tag{3.24}$$

These boundary conditions follow from the following no-slip conditions and symmetry of flow

$$\left.\begin{aligned}&w=0 \text{ as } y=0\Rightarrow g(0)=0,\\&w=w_{\varpi}(t) \text{ as } y=1\Rightarrow g(1)=1,\\&v=0 \text{ as } y=1\Rightarrow g'(1)=0,\\&v_{\xi}(0)=0 \text{ as } y=0\Rightarrow g''(0)=0,\\&T=T_{\varpi} \text{ as } y=0\Rightarrow \theta(0)=1,\\&T\Rightarrow T_{\infty} \text{ as } y=1\Rightarrow \theta(1)=0\end{aligned}\right]\tag{3.25}$$

Here, R and Q both are functions of t, used for similarity solution as a constant, ρ is the density of fluid. Moreover, there arise two cases from the definition of $s(t)$ in first Case when $R>0$ implies that plates are moving away from each other and when $R<0$ implies that plates are moving towards each other.

4.1 Solution Procedure With VIM

To apply Variational iteration method, initial approximations are taken as follows:

$$g_0(\xi)=\frac{3}{2}\xi-\frac{1}{2}\xi,\ \theta_0(\xi)=\frac{3}{2}\xi+\frac{1}{2}\xi,\tag{4.1}$$

According to VIM the Lagrange multiplier is given by as follows:

$$g_{n+1}(\xi)=g_n(\xi)+\int_0^{\xi}\lambda_g(s)\left[R\left[g_n^{(iv)}-M^2 g_n''-gg_n'''+g_n'g_n''+\left(2+\frac{1}{\rho}\right)g_n''+\xi g_n'''\right]\right]ds,\tag{4.2}$$

$$\theta_{n+1}(\xi)=\theta_n(\xi)+\int_0^{\xi}\lambda_\theta(s)\left[\theta_n''-\left(-\xi+g_n\right)\theta_n'\frac{\Pr R}{\rho}+\Pr Ec g_n''^2\right]ds,\tag{4.3}$$

To find the Lagrange multipliers $\lambda_g(s)$ and $\lambda_\theta(s)$, we first restrict the non-linear terms and then apply the correctional functional δ on both sides, we obtain the following results:

$$\lambda_g(s)=\frac{-(\xi-s)^3}{3!},\ \lambda_\theta(s)=\xi-s,\tag{4.4}$$

Using Equation (4.4) in (4.2) –(4.3) respectively, we have

$$g_{n+1}(\xi)=g_n(\xi)+\int_0^{\xi}\frac{-(\xi-s)^3}{3!}\left[R\left[g_n^{(iv)}-M^2 g_n''-gg_n'''+g_n'g_n''+\left(2+\frac{1}{\rho}\right)g_n''+\xi g_n'''\right]\right]ds,\tag{4.5}$$

$$\theta_{n+1}(\xi)=\theta_n(\xi)+\int_0^{\xi}\left(\xi-s\right)\left[\theta_n''-\left(-\xi+g_n\right)\theta_n'\frac{\Pr R}{\rho}+\Pr Ec g_n''^2\right]ds,\tag{4.6}$$

The general solution of the above equation is given as follows:

$$g(\xi) = \lim_{n\to\infty} g_n$$
$$\theta(\xi) = \lim_{n\to\infty} \theta_n. \quad (4.7)$$

4.2 Solution Procedure With VPM

To implement VPM, initial approximations for governing nonlinear ODEs are as follows

$$g_0(\xi) = \frac{3}{2}\xi - \frac{1}{2}\xi,\ \theta_0(\xi) = \frac{3}{2}\xi + \frac{1}{2}\xi, \quad (4.8)$$

According to VIM the Lagrange multiplier is given by as follows:

$$g_{n+1}(\xi) = g_n(\xi) + \int_0^{\xi} \lambda_g(s)\left[R\left[M^2 g_n'' + g g_n''' - g_n' g_n'' - \left(2 + \frac{1}{\rho}\right) g_n'' - \xi g_n'''\right]\right] ds, \quad (4.9)$$

$$\theta_{n+1}(\xi) = \theta_n(\xi) + \int_0^{\xi} \lambda_\theta(s)\left[\left(-\xi + g_n\right)\theta_n' \frac{\Pr R}{\rho} - \Pr Ec g_n''^2\right] ds, \quad (4.10)$$

To find the Lagrange multipliers $\lambda_g(s)$ and $\lambda_\theta(s),$ we first restrict the non-linear terms and then apply the correctional functional δ on both sides, we obtain the following results:

$$\lambda_g(s) = \frac{-(\xi - s)^3}{3!},\ \lambda_\theta(s) = \xi - s, \quad (4.11)$$

Using Equation 4.31 in 4.29 and 4.30 respectively, we have

$$g_{n+1}(\xi) = g_n(\xi) + \int_0^{\xi} \frac{-(\xi - s)^3}{3!}\left[R\left[M^2 g_n'' + g g_n''' - g_n' g_n'' - \left(2 + \frac{1}{\rho}\right) g_n'' - \xi g_n'''\right]\right] ds, \quad (4.12)$$

$$\theta_{n+1}(\xi) = \theta_n(\xi) + \int_0^{\xi} \left(\xi - s\right)\left[\left(-\xi + g_n\right)\theta_n' \frac{\Pr R}{\rho} - \Pr Ec g_n''^2\right] ds, \quad (4.13)$$

The general solution of the above equation is given as follows:

$$g(\xi) = \lim_{n\to\infty} g_n$$
$$\theta(\xi) = \lim_{n\to\infty} \theta_n. \quad (4.14)$$

5. Results and Discussion

The numerical results illustrate the significant influence of various parameters on the flow and thermal characteristics of the system. An increase in the parameter R leads to a monotonic rise in the normal velocity (Figure. 1), whereas it causes a simultaneous reduction in the longitudinal velocity (Figure. 2), a trend similarly observed for the parameter ρ as depicted in Figure. 3. Regarding the thermal profile, the Prandtl number Pr exerts a cooling effect, where higher values of Pr result in a notable decrease in temperature (Figure. 4). This cooling behavior is also evident in the temperature fields influenced by parameters shown in Figures. 5 and 6. Furthermore, the analysis reveals that both the Eckert number Ec and the parameter ρ exert a comparable influence on the temperature distribution (Figure. 7), highlighting their consistent roles in governing the heat transfer mechanism within the boundary layer.

The observed reduction in longitudinal velocity, especially with respect to the parameter ρ, indicates the important influence of the magnetic field (MHD) on the porous material. Together, these components provide a strong resistive force that thickens the momentum boundary layer and resists fluid motion. This force is made up of the Lorentz force and Darcy resistance. The interaction between the conducting fluid and the magnetic field produces the Lorentz force, which serves as a "braking" mechanism. The rate of stretching-induced transport over the surface is similarly slowed by the internal drag (Darcy impedance) produced by the porous matrix, which further slows the fluid particles.

Figure. 3 shows a similar inhibitory effect for the parameter ρ. The decrease in velocity suggests that the presence of the porous medium and the magnetic field (MHD) produces a resistive force that thickens the momentum boundary layer and opposes fluid motion. This force is commonly known as the Lorentz force and Darcy resistance. The trends in Figure. 5 and 6, where certain fluid parameters restrict heat penetration into the bulk flow—a critical factor for applications requiring quick heat dissipation—further support this cooling behavior. Moreover, both methods converge to the exact solution, the **VPM** demonstrated a slightly faster convergence for the non-linear velocity components due to its specific handling of the embedding parameter.

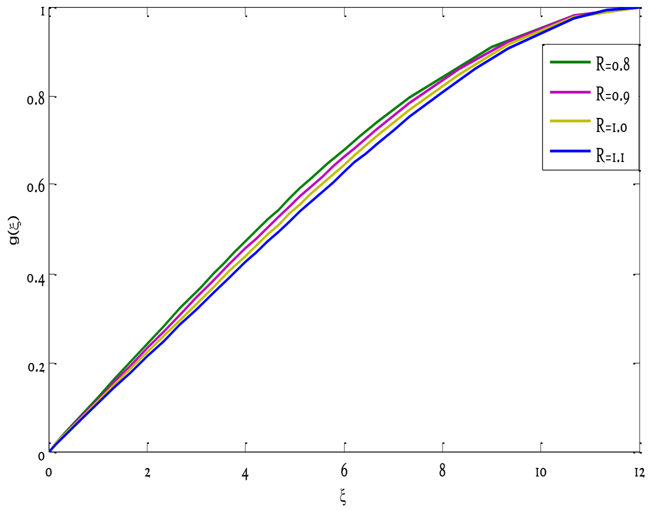

Figure.1*: Influence of R on velocity in y direction*

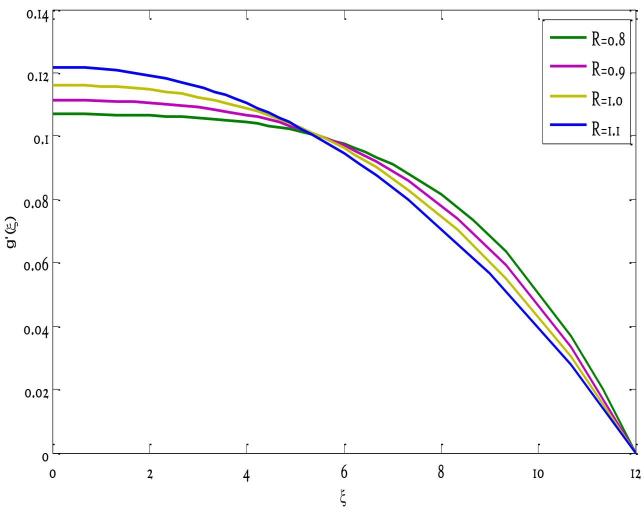

Figure.2*: Influence of R on velocity in x-direction*

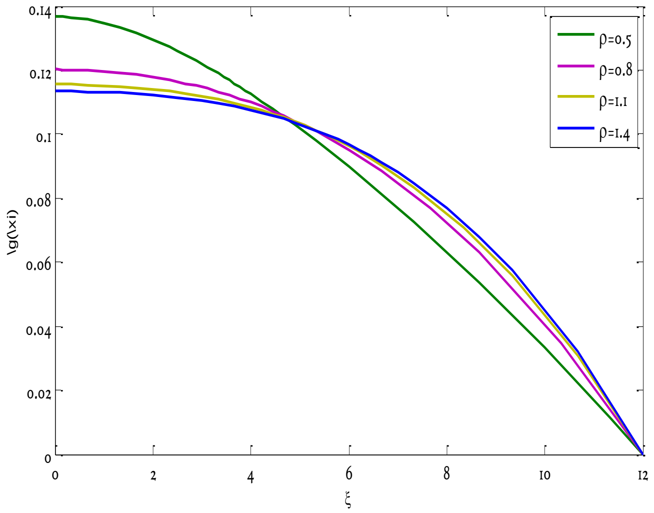

Figure.3: Influence of ρ on velocity in x-direction

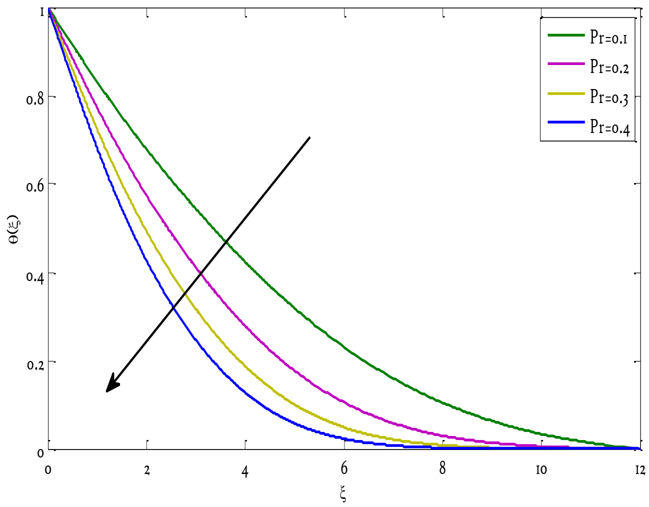

***Figure.4:** Influence of Pr on temperature field*

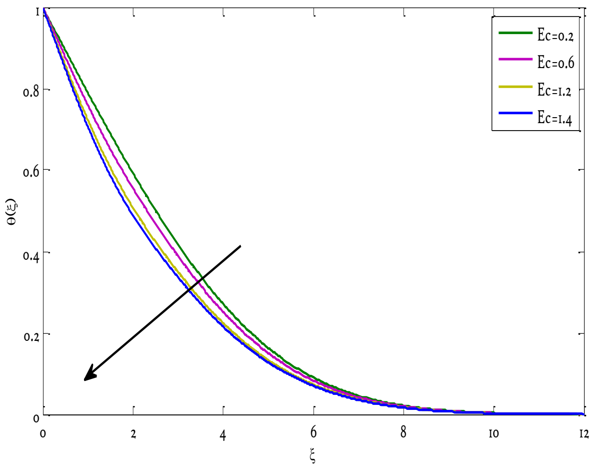

Figure 5.: *Influence of Ec on temperature field*

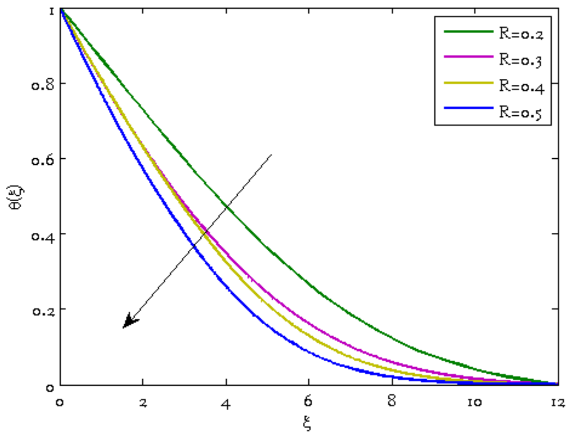

Figure.6: *Influence of R on temperature field*

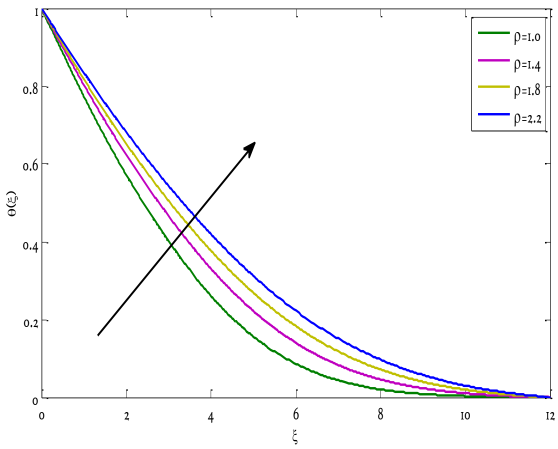

Figure.7: *Influence of ρ on temperature field*

6. Conclusion

The fluid flow of magneto-hydrodynamic (MHD) Burgers flow over an inclined stretched sheet through porous media is effectively analyzed in this paper. The complicated governing differential equations were solved with great accuracy by applying computationally efficient analytical approaches, namely the Variational Iteration Method (VIM) and Variational Perturbation Method (VPM). The analysis shows that the velocity and temperature fields are quite sensitive to physical characteristics: longitudinal velocity shows a definite inhibitory response, but normal velocity shows a monotonic rise with certain flow parameters. Additionally, the thermal study emphasizes the importance of viscous dissipation in heat transmission by showing a notable improvement in the temperature profile caused by increasing values of the Eckert number Ec.

Key Findings are

- With increase of with increase of R normal velocity increases monotonically and longitudinal velocity decreases.
- The effect of Pr, Ec and R is similar on temperature profile.
- Increasing the value of ρ causes increase in temperature profile.

These findings not only validate the robustness of the employed analytical methods but also offer critical insights into the optimization of heat and mass transfer processes in industrial engineering and advanced fluid mechanics. In conclusion, a complex framework for fluid analysis is introduced by the combination of the Burgers fluid model and the porous medium under an inclined geometry. Future researchers will have a solid mathematical foundation to investigate more intricate non-Newtonian models thanks to the validation of the VIM and VPM approaches. In the end, these findings offer a tactical road map for improving the effectiveness of fluid transport mechanisms and thermal systems in contemporary industrial engineering.

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