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Soliton Solutions of Variant Boussinesq Equations Through Exp-function Method

K. Ayub, M. Saeed, M. Ashraf, M. Yaqub and M. Hassan

***Abstract*— This research article presents soliton solutions of Variant Boussinesq equations. The Boussinesq equation governs the dynamics of shallow water waves that are seen in various places like sea beaches, lakes and rivers. By a suitable transformation the nonlinear partial differential equation is converted into nonlinear ordinary differential equation. The exp-function method is applied to solve the mathematical problem. The novel type results based on the solitary wave structures contributes a lot in the regime of nonlinear wave phenomena. It is observed that scheme is highly trustworthy and may be extended to other nonlinear models represented in the form of highly nonlinear differential equations.**

***Index Terms*— Soliton solutions, Exp-function technique, Variant Boussinesq equations, Maple 18**.

I. INTRODUCTION

LARGE assortments of physical, chemical and biological singularities are ruled by nonlinear partial differential equations. One of the most stimulating progresses of nonlinear sciences and mathematical physics has been the development of methods to look for exact solutions of nonlinear partial differential equations. Exact solutions to nonlinear partial differential equations play a vital role in nonlinear sciences, especially in nonlinear physical sciences. Since they can provide much physical information and more understanding into the physical features of the problem, they lead to further applications.

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The rapid development of nonlinear sciences looks over a wide range of trustworthy and well-organized techniques which are of great help in tackling physical problems even of highly complicated nature. After the observation of solitary phenomena by John Scott Russell [1] in 1844 and the Korteweg-de Vries (KdV) equation was solved by Gardner et al. [2] by the inverse scattering method, finding the exact solutions of nonlinear evolution equations (NLEEs) has turned out to be one of the most stirring areas of research. The solitary wave solutions appear commonly in nature; kink-shaped tanh-solutions and bell-shaped sech-solutions, models the wave phenomena in elastic media, plasmas, solid state physics, condensed matter physics, electrical circuits, optical fibers, chemical kinematics, fluids, bio-genetics etc. In renowned example, traveling wave solutions are obtained by KdV equation that describes water waves. Many different methods are used for the exact solutions of these equations. Aside from their physical bearing, if the closed-form solutions of NLEEs are available, it facilitates the numerical solvers in comparison, and assist in the stability analysis. In soliton theory, many methods and techniques are available to deal with the problem for finding solitary wave solutions for NLEEs such as the Backlund transformation method [3], the variational iteration method [4], the homogeneous balance method, the tanh-function method [5-7], the F-expansion method], the first integration method, the exp-function method [8], the truncated Painleve expansion method, the Weierstrass elliptic function method [9] and the Jacobi elliptic function expansion method. For integrateable nonlinear differential equations [10], the inverse scattering transform method [11] and the Hirota method [12] are used for searching the exact solutions. Some other systematical solution approaches include, the invariant subspace method [13]. The expansion around solutions to a Riccati equation is discussed in [14]. Recent results on exact solutions and integral transform methods are presented in [15-20].

The manuscript constructs exact traveling wave solutions to a nonlinear Boussinesq equation. Exp-function methods are special cases of the transformed rational function method [21], or more generally the multiple exp-function expansion method [22-23]. The transformed rational function method delivers a more systematical and appropriate treatment of the soliton solution process, unifying the tanh-function method, the sech-function method, the homogeneous balance method, the extended tanh-function method, the sine–cosine method, the coth- function method, the Jacobi elliptic function method, the exp-function method, the F-expansion type method, the mapping method and the extended F-expansion method. Its significant point is to hunt for rational solutions based upon rational function transformations. Based on Backlund transformation theory and the Fourier theory, modified and improved modified exp-function techniques are the most general methods, in the literature so far, in generating travelling wave solutions and multiple wave solutions respectively. That means that all traveling wave solutions could be presented by the transformed rational function method and all multiple wave solutions such as N-soliton solutions could be presented by the multiple exp-function method, which also generalizes the exp-function method. A direct search for exact solutions to the nonlinear differential equation is made under the same expansion idea [24]. Solitary wave solutions could be generated by the Hirota bilinear method and generalized bilinear techniques [25]. The subsequent works [26-34] have shown the complete reliability and efficiency of the algorithms.

The basic motivation of this paper is the extension of the applications of exp-function methods [35-37] for the solutions of nonlinear Boussinesq equation by finding the solitary and periodic solutions. In 1872 paper of Boussinesq had introduced the equations now known as the Boussinesq equations. This equation has many physical applications like analysis of long shallow water waves and percolation of water in porous surface. In fluid dynamics, the Boussinesq approximation for water waves is an approximation valid for weakly non-linear and fairly long waves. The approximation is named after Joseph Boussinesq, who first derived them in response to the observation by John Scott Russell of the wave of translation also known as solitary wave or soliton. The dynamics of shallow water waves, that are seen in various places like sea beaches, lakes and rivers, are governed by the Boussinesq Equation (BE). The Korteweg–deVries (KdV) equation that models shallow water waves is definitely very well known. However, the BE gives a much better approximation to such waves. There are two forms of the Boussinesq Equation BE, and both are with cubic nonlinearity. The soliton solutions will be extremely useful in carrying out further analysis in the context of shallow water waves that arises in the context of oceanography.

The soliton solution method under study is quite compatible and user friendly for such nonlinear problems. Analytical results are very boosting. The solution procedure of this technique is quite simple, explicit, and easily be extended to all types of NLEEs [38-40]. Proposed technique is divided in different parts. In next section, analysis of method used to attain soliton wave solutions is presented. Third section is devoted to application of exp-function technique. Results and their discussion are given in section IV to draw some conclusions.

II. Analysis of Technique

Consider the general nonlinear partial differential equation of the type:

$$P(\Psi, \Psi_t, \Psi_x, \Psi_y, \Psi_{xx}, \Psi_{yy}.....) = 0. \tag{1}$$

Invoking a transformation:

$$\xi = kx + my + nz + \Omega t. \tag{2}$$

In above mentioned transformation $m, \Omega. \neq 0.$

Partial differential equation (1) changed into ODE given as:

$$Q(\Psi, \Psi', \Psi'', \Psi''',.....) = 0. \tag{3}$$

Dash on Ψ indicates derivative with respect to ξ in above equation.

The assumed soliton wave solution in exp-function technique:

$$\Psi(\xi) = \frac{\sum_{i=-e}^{f} a_i \exp[i\xi]}{\sum_{j=-r}^{s} b_j \exp[j\xi]}. \tag{4}$$

Where r, e, s and f are the positive integers, a_i and b_j are constants. Equation (4) can be rewritten in the form as below.

$$\Psi(\xi) = \frac{a_e \exp(e\xi) + ... + a_{-f} \exp(-f\xi)}{b_r \exp(r\xi) + ... + b_{-s} \exp(-s\xi)}. \tag{5}$$

The outcome of equation (5) leads to solitary wave solutions of the model equation. Calculating values by using [25], finally results in $r = e, s = f.$ (6)

III. Solution Procedure

Consider the Variant Boussinesq equations

$$\begin{aligned} &\Psi_t + \Phi_y + \Psi\Psi_y = 0, \\ &\Phi_t + \left(\Psi\Phi\right)_y + \Psi_{yyy} = 0. \end{aligned} \tag{7}$$

Using equation (2) Variant Boussinesq equation can be converted to an ordinary differential equation:

$$\Omega\Psi' + m\Phi' + m\Psi\Psi' = 0, \tag{8}$$

$$\Omega\Phi' + m(\Psi\Phi)' + m^3\Psi''' = 0. \tag{9}$$

Integrating equation (8)

$$\Phi = -\frac{\Omega}{m}\Psi - \frac{1}{2}\Psi^2 + c_1. \tag{10}$$

In above equation c_1 is integration constant.

Substituting value from equation (10) into equation (9) yields:

$$\Omega\left(-\frac{\Omega}{m}\Psi - \Psi\Psi'\right) + m\left[\Psi\left(-\frac{\Omega}{m}\Psi - \frac{1}{2}\Psi^2 + c_1\right)\right]' + m^3\Psi''' = 0. \tag{11}$$

Soliton solution of Variant Boussinesq equation is in the form of equation (4).

The final soliton solutions does not depend on selection of values of e, f and r, s.

Case I: In first case take $r = e = 1$ and $s = f = 1$.

$$\Psi(\xi) = \frac{a_1 \exp[\xi] + \ldots + a_{-1}\exp[-\xi]}{b_1 \exp[\xi] + \ldots + b_{-1}\exp[-\xi]}. \tag{12}$$

Substituting equation (12) into equation (11), we have

$$\frac{1}{A}\begin{bmatrix} e_5 \exp(5\xi) + e_4 \exp(4\xi) + \\ e_3 \exp(3\xi) + e_2 \exp(2\xi) + \\ e_1 \exp(\xi) + e_0 + e_{-1}\exp(-\xi) \\ + e_{-2}\exp(-2\xi) + e_{-3}\exp(-3\xi) + \\ e_{-4}\exp(-4\xi) + e_{-5}\exp(-5\xi) \end{bmatrix} = 0. \tag{13}$$

Where $A = (b_1 \exp(\xi) + b_0 + b_{-1}\exp(-\xi))^5$, and e_i can be obtained with the help of Maple 18. Putting coefficients of $\exp(i\xi)$ equal to zero:

$$\begin{aligned} &e_{-1} = 0, e_{-2} = 0, e_{-3} = 0, \\ &e_{-4} = 0, e_{-5} = 0, e_0 = 0, e_1 = 0, . \\ &e_2 = 0, e_3 = 0, e_4 = 0, e_5 = 0 \end{aligned} \tag{14}$$

Soliton solutions satisfying Variant Boussinesq equations are given below.

1[st] Solution set:

$$\left\{ \begin{aligned} &c_1 = c_1, \Omega = \Omega, a_{-1} = 0, a_0 = a_0, \\ &a_1 = a_1, b_{-1} = 0, b_0 = b_0, b_1 = \frac{b_9 a_1}{a_0} \end{aligned} \right\}$$

The solitary solution $\Psi(y,t)$ is given as

$$\Psi(y,t) = \frac{a_{-1}e^{-(my+\Omega t)} + a_1 e^{(my+\Omega t)}}{b_{-1}e^{-(my+\Omega t)} + \frac{b_{-1}a_1}{a_{-1}} e^{(my+\Omega t)}}.$$

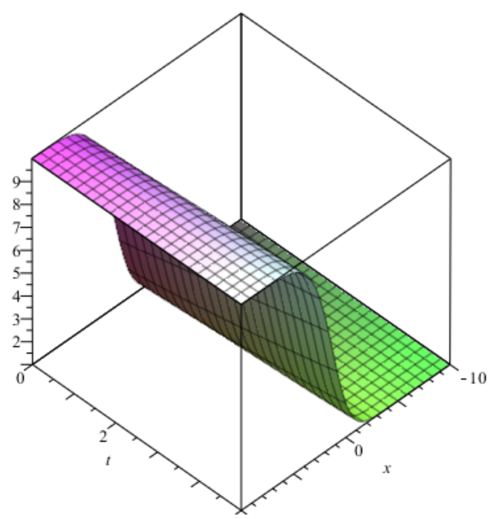

Fig. 1: Soliton solution for a_1=1, b_{-1}=1, a_{-1}=1, Ω=2, m=1.

2[nd] Solution set:

$$\left\{ \begin{aligned} &c_1 = c_1, \Omega = \Omega, a_{-1} = a_{-1}, a_0 = 0, a_1 = a_1, \\ &b_{-1} = b_{-1}, b_0 = 0, b_1 = \frac{b_{-1}a_1}{a_{-1}} \end{aligned} \right\}$$

Soliton solution $\Psi(y,t)$ is in the following form

$$\Psi(y,t)=\frac{\frac{b_{-1}a_0}{b_0}e^{(my+\Omega t)}}{b_{-1}e^{-(my+\Omega t)}+b_0+b_1e^{(my+\Omega t)}}+$$

$$\frac{a_0-\dfrac{\left(\dfrac{\left(-6b_{-1}b_0b_1-b_0^3\right)b_{-1}a_0}{b_0}+b_1b_{-1}^2a_0+b_{-1}a_0b_0^2\right)}{5b_0b_{-1}^2}e^{(my+\Omega t)}}{b_{-1}e^{-(my+\Omega t)}+b_0+b_1e^{(my+\Omega t)}}.$$

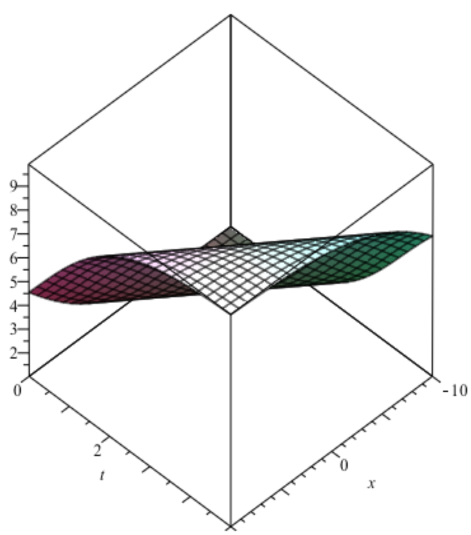

Fig. 2. Soliton solution for $b_0=1$, $b_1=1$, $a_0=1$, $b_{-1}=1$, Ω, m=1.

3rd Solution set:

$$\left\{\begin{array}{l} c_1=c_1,\Omega=\Omega,a_{-1}=\dfrac{b_{-1}a_0}{b_0},a_0=a_0,\\ a_1=\dfrac{\left(\dfrac{\left(-6b_{-1}b_0b_1-b_0^3\right)\times b_{-1}a_0}{b_0}+b_1b_{-1}^2a_0+b_{-1}a_0b_0^2\right)}{5b_0b_{-1}^2},\\ b_{-1}=b_{-1},b_0=b_0,b_1=b_1 \end{array}\right\}$$

Therefore solitary wave solution $\Psi(y,t)$ given below

$$\Psi(y,t)=\frac{a_0+a_1e^{(my+\Omega t)}}{b_0+\dfrac{b_0a_1}{a_0}e^{(my+\Omega t)}}$$

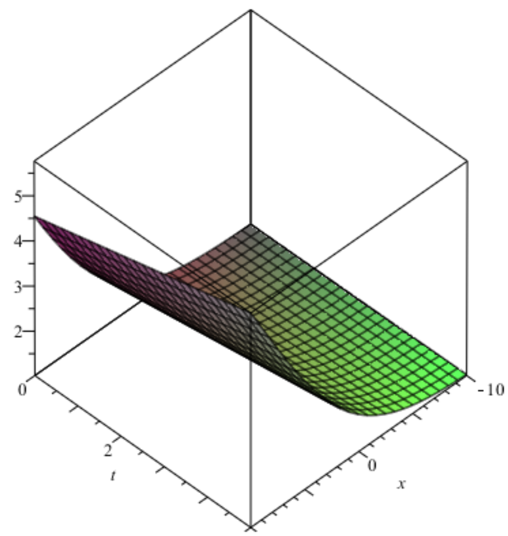

Fig. 3. Soliton solution for $b_0=1$, $a_1=1$, $a_0=1$, Ω=1, m=1

Case II. In this case take $r=e=2$ and $s=f=1$, the trial solution takes the form:

$$\Psi(\xi)=\frac{a_2\exp[2\xi]+a_1\exp[\xi]+a_0+a_{-1}\exp[-\xi]}{b_2\exp[2\xi]+b_1\exp[\xi]+b_0+b_{-1}\exp[-\xi]}. \quad (15)$$

1st Solution set:

$$\left\{\begin{array}{l} c_1=-\dfrac{a_0(2mb_0+a_0)}{2b_0^2},\Omega=-\dfrac{m(mb_0+a_0)}{b_0},\\ a_{-1}=\dfrac{b_{-1}(mb_0+a_0)}{b_0},a_0=a_0,a_1=0,a_2=0\\ b_{-1}=b_{-1},b_0=b_0,b_1=0,b_2=0 \end{array}\right\}$$

Therefore attained solution $\Psi(y,t)$ can be written as:

$$\Psi(y,t)=\frac{\dfrac{b_{-1}a_1}{b_1}e^{-(my+\Omega t)}+\dfrac{b_0a_1}{b_1}+a_1e^{(my+\Omega t)}}{b_{-1}e^{-(my+\Omega t)}+b_0+b_1e^{(my+\Omega t)}}$$

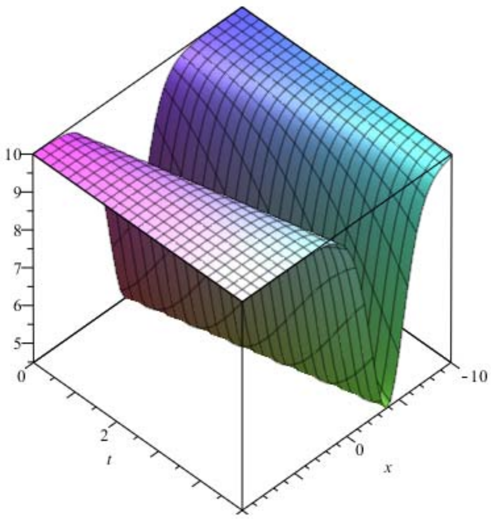

Fig. 4. Soliton solution for b_0=1, b_1=1, a_1=1, b_{-1}=1, Ω=1, m=1

2nd Solution set:

$$\left\{\begin{array}{l} c_1=\dfrac{a_0\left(2mb_0-a_0\right)}{2b_0^2},\Omega=\dfrac{m\left(mb_0-a_0\right)}{b_0}, \\ a_{-1}=-\dfrac{b_{-1}\left(mb_0+a_0\right)}{b_0},a_0=a_0,a_1=0,a_2=0 \\ b_{-1}=b_{-1},b_0=b_0,b_1=0,b_2=0 \end{array}\right\}$$

Hence the solitary solution $\Psi(y,t)$ can be written as:

$$\Psi(y,t)=\frac{\dfrac{b_{-1}a_2}{b_2}e^{-(my+\Omega t)}+\dfrac{b_0a_2}{b_2}+\dfrac{b_1a_2}{b_2}e^{(my+\Omega t)}+a_2e^{2(my+\Omega t)}}{b_{-1}e^{-(my+\Omega t)}+b_0+b_1e^{(my+\Omega t)}+b_2e^{2(my+\Omega t)}}.$$

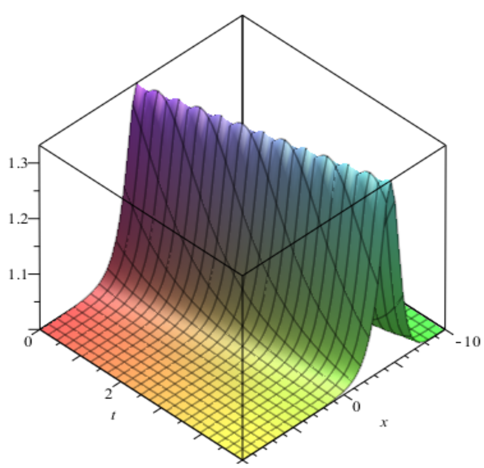

Fig. 5. Soliton solution for b_0=1, b_{-1}=1, b_1=1, b_2=1, a_2=1, Ω=1, m=1.

3rd Solution set:

$$\left\{\begin{array}{l} c_1=-\dfrac{\left(2m^2b_1^2+a_1^2\right)}{2b_1^2},\Omega=-\dfrac{ma_1}{b_1}, \\ a_{-1}=-\dfrac{a_1a_0^2}{16m^2b_1^2},a_0=a_0,a_1=a_1,a_2=0 \\ b_{-1}=-\dfrac{a_0^2}{16m^2b_1},b_0=0,b_1=b_1,b_2=0 \end{array}\right\}$$

The obtained generalized solitary solution $\Psi(y,t)$ is given as

$$\Psi(y,t)=\frac{-\dfrac{a_1a_0^2}{16m^2b_1^2}e^{-(my-\Omega t)}+a_0+a_1e^{(my-\Omega t)}}{-\dfrac{a_0^2}{16m^2b_1}e^{-(my-\Omega t)}+b_1e^{(my-\Omega t)}}$$

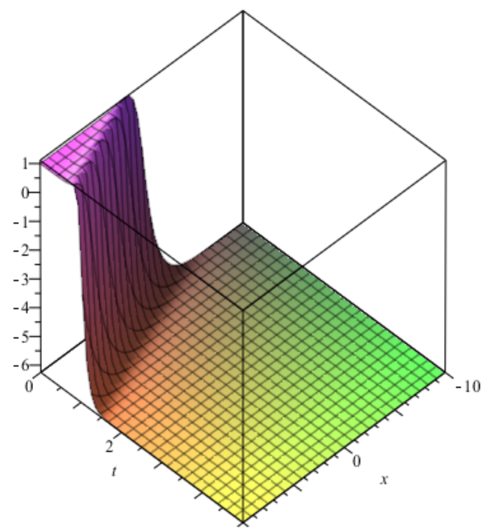

Fig. 6. Soliton solution for a_0=1, b_1=1, a_1=1, Ω=1, m=1.

IV. Results and Discussion

The soliton wave formation by solving nonlinear Variant Boussinesq equations has been examined via a novel analytical technique, exp-function method. The findings are mentioned and discussed as follows.

A soliton is a solitary wave which promulgate deprived of variation in shape. The cause of arising soliton is the indirect balance among dispersive and nonlinear effects. Soliton wave amplitudes and velocities are handled by different factors. Figures indicates graphical solutions for altered values of parameters. Figures depict graphical representation of nonlinear Variant Boussinesq equations for mentioned values of parameters. Since solitary wave solutions be influenced by arbitrary functions. So it is concluded, different constraints can be selected as input to our simulations. The graphical representations in Figs. (1-6) signifies solitary waves for various values of parameters. In both cases it is observed that soliton wave solutions does not strongly depends on values of parameters, and equivalent solitary wave's solutions are attained.

V. CONCLUSIONS

In this paper, novel soliton wave formation of Variant Boussinesq equations is developed. It is observed that nonlinear differential equations permit soliton type solutions. The applied algorithm is very beneficial to validate the results attained by the exact solution. The intimacy among the outcomes reveals that it is a powerful tool for solving differential equations. In solitary wave theory, Variant Boussinesq equations has a significant role as it discloses soliton wave solutions of various nature, and exp-function technique is a decent and reliable tool to handhold these equations. It is concluded that proposed method is an efficient technique to check the physical behavior of solitary waves analytically. Moreover, exp-function method has wide applications due to less computational work. This scheme is capable and effective for estimating solitary wave solutions of nonlinear mathematical models. The attained results show that the developed technique is very inspiring and trustworthy for handling different classes of non-linear evolutions equations. The graphical results also show the soliton waves of various types.

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